Physics 6A

Practice Midterm #2 solutions
1. A locomotive engine of mass $M$ is attached to 5 train cars, each of mass $M$. The engine produces a constant force that moves the train forward at acceleration $a$. If 3 of the cars are removed, what will be the acceleration of the shorter train?

We just use Newton’s 2\textsuperscript{nd} law here: Net force = (total mass) x (acceleration)

Initially, the total mass is 6$M$ (5 cars, plus the engine)

After removing 3 cars the total mass is 3$M$ (2 cars, plus the engine)

In both cases the Net force is the same (the engine didn’t change).

Here is the formula in both cases:

\begin{align*}
\text{Initial} & \quad F = (6M)(a) \\
\text{Final} & \quad F = (3M)(a_{\text{final}}) \\
\end{align*}

Setting these equal gives $a_{\text{final}} = 2a$

Another way to think about this one is that since the mass is cut in half, the acceleration must double.
2. Two boxes are placed next to each other on a smooth flat surface. Box A has mass 1 kg and Box B has mass 3 kg. A constant horizontal force of 8 N is applied to Box A. Find the force exerted on Box B.

We can think of this as a single box with total mass 4kg. Then using $F = ma$ we get the acceleration of the whole system.

\[ 8N = 4 \text{kg} \cdot a_{\text{system}} \]

\[ a_{\text{system}} = 2 \frac{m}{s^2} \]

Now we do the same thing, but just for box B:

\[ F_B = 3 \text{kg} \cdot 2 \frac{m}{s^2} \]

Remember – Box B has a mass of only 3kg

\[ F_B = 6N \]
3. Two boxes are placed next to each other on a flat surface. Box A has mass 1kg and Box B has mass 3kg. The coefficients of friction are 0.3 and 0.4 for kinetic and static friction, respectively. A constant horizontal force of 8N is applied. Find the acceleration of Box B.

We can think of this as a single box with total mass 4kg.
Then using \( F = ma \) we get the acceleration of the whole system.
We need to account for friction, so first find the maximum force of static friction:

\[
F_{\text{static, max}} = \mu_s \cdot N
\]

\[
F_{\text{static, max}} = (0.4) \cdot \left(4 \text{kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2}\right) = 15.68 \text{N}
\]

This friction force is more than the 8N force trying to move the boxes, so they are held in place by friction. Thus the acceleration of both boxes is 0.

*Note that the actual force of friction holding the boxes in place is only 8N – just enough to keep them from moving.*
4. A crate filled with books rests on a horizontal floor. The total weight of the crate and books is 700\text{N}. The coefficients of friction are 0.35 for kinetic and 0.45 for static. A force of 450\text{N} is applied to the crate at an angle of 20° below the horizontal. Find the acceleration of the crate.

We need to find the components of the 450\text{N} force:

\[
F_x = 450\text{N} \cdot \cos(20°) \approx 423\text{N}
\]

\[
F_y = 450\text{N} \cdot \sin(20°) \approx 154\text{N}
\]

We need to find the normal force, so we can determine whether static friction will hold the crate in place. For this, we use the fact that the forces in the y-direction cancel out:

\[
F_{\text{net},y} = F_{\text{normal}} - 700\text{N} - 154\text{N} = 0
\]

\[
F_{\text{normal}} = 854\text{N}
\]

This gives a maximum force of static friction = (0.45)(854\text{N}) = 384\text{N}

Comparing this with the x-component of the applied force, we see that this is not enough friction to hold the crate in place, so the actual friction force will be kinetic.

\[
F_{\text{friction},k} = (0.35)(854\text{N}) = 299\text{N}
\]

Now we can use \( F = ma \) again for the x-direction forces:

\[
F_{\text{net},x} = F_x - F_{\text{friction},k} = ma \Rightarrow 423\text{N} - 299\text{N} = \left( \frac{700\text{N}}{9.8 \text{ m/s}^2} \right) = a \Rightarrow a = 1.74 \text{ m/s}^2
\]
5. Blocks A and B are connected by a massless string and placed as shown, with Block A on the horizontal surface at the top and Block B on the downward slope of the 60° incline. The string passes over a frictionless, massless pulley. The inclines are frictionless, but the horizontal surface has coefficient of kinetic friction 0.45. Block A has mass 5kg. The system accelerates at 4 m/s². Find the mass of Block B.

Here is the force diagram for this problem. The blocks will move together, so we can consider the motion of the entire system in one formula. Our axis system will be chosen to coincide with the string.

With this choice, our formula is: 

\[-F_k + m_B g \sin \theta = (m_A + m_B) \cdot a_{system}\]

Notice that the tension forces cancel out. We are assuming the string and pulley are massless, so the tension is the same throughout the string. Thus the forward tension T on A is canceled by the backward tension T on B. Or we say that the tensions are internal forces, and thus do not affect the acceleration of the entire system.
5. Blocks A and B are connected by a massless string and placed as shown, with Block A on the horizontal surface at the top and Block B on the downward slope of the 60° incline. The string passes over a frictionless, massless pulley. The inclines are frictionless, but the horizontal surface has coefficient of kinetic friction 0.45. Block A has mass 5kg. The system accelerates at 4 m/s². Find the mass of Block B.

\[
F_k + m_B g \sin \theta = (m_A + m_B) \cdot a_{\text{system}}
\]

Now we need to find the friction force. For this we need the Normal force on A. From the diagram we see that \(N_A = m_A g\).

Since the blocks are in motion, we are dealing with kinetic friction.

\[
F_k = \mu_k \cdot m_A g = (0.45)(5kg \cdot 9.8 \frac{m}{s^2}) = 22.05N
\]

Now we can substitute into the force formula:

\[
(22.05N) + m_B (9.8 \frac{m}{s^2}) \sin(60°) = (5kg + m_B) \cdot (4 \frac{m}{s^2})
\]

A bit of algebra will get us the desired mass \(m_B\)

\[
-22.05N + m_B (8.487) = 20N + (4)m_B
(4.487)m_B = 42.05N
m_B = 9.37kg
\]
6. A 0.2 kg piece of wood is being held in place against a vertical wall by a horizontal force of 5 N. Find the magnitude of the friction force acting on the wood.

From the force diagram, we can see that the friction force must equal the weight of the piece of wood to keep it from falling.

\[ F_{\text{friction}} = mg \]
\[ F_{\text{friction}} = (0.2\text{kg})(9.8 \text{ m/s}^2) \]
\[ F_{\text{friction}} = 1.96\text{N} \]
7. A 4 kg block on a horizontal surface is attached to a spring with a force constant of 50 N/m. As the spring and block are pulled forward at constant speed, the spring stretches by 25 cm. Find the coefficient of kinetic friction between the block and the table.

The key phrase here is “constant speed”. Since the block is moving at constant speed (and direction) we know that its acceleration is 0. Thus the net force is 0 as well.

Here is the force diagram. Since the net force is 0, we know that the Normal force must equal the weight, and the Friction force must equal the Spring force.

The spring force obeys Hooke’s Law:

\[ F_{\text{spring}} = k \cdot x = \left( 50 \frac{\text{N}}{\text{m}} \right) \cdot (0.25 \text{m}) = 12.5 \text{N} \]

The weight is:

\[ mg = (4 \text{kg}) \left( 9.8 \frac{\text{m}}{\text{s}^2} \right) = 39.2 \text{N} \]

Finally, we can put this together to find the friction:

\[ F_{\text{spring}} = F_{\text{friction}} = \mu_k \cdot N \]

\[ 12.5 \text{N} = \mu_k \cdot (39.2 \text{N}) \]

\[ \mu_k = 0.32 \]
8. A 1000 kg car is driven around a turn of radius 50 m. What is the maximum safe speed of the car if the coefficient of static friction between the tires and the road is 0.75?

The friction force must be directed toward the center of the circle. Otherwise the car will slide off the road. If we want the maximum speed, then we want the maximum static friction force. The road is flat (not banked), so the Normal force on the car is just its weight.

\[ F_{\text{friction, static, max}} = \mu_s \cdot mg = (0.75)(1000 \text{kg})(9.8 \frac{\text{m}}{\text{s}^2}) = 7350 \text{N} \]

This friction force is the only force directed toward the center, so it must be the centripetal force:

\[ F_{\text{friction}} = F_{\text{centripetal}} = \frac{mv^2}{r} \]

\[ 7350 \text{N} = \frac{(1000 \text{kg})(v^2)}{50 \text{m}} \]

\[ v = 19.2 \frac{\text{m}}{\text{s}} \]
9. A 90 kg man drives his car at a constant speed of 15 m/s over a small hill that has a circular cross section of radius 40 m. Find his apparent weight as he crests the top of the hill. (Hint: the apparent weight is the same as the normal force on the man.)

When the car reaches the top of the hill, it will have only 2 forces: its weight, and the normal force supplied by the road. Since the man is sitting in the car, he feels a normal force as well.

The car (and man) must be accelerating toward the center of the circle, so the net force on the man will be equal to the centripetal force required to keep him moving along the circle.

\[ \sum F_{\text{cent}} = mg - \text{Normal} = \frac{mv^2}{r} \]

Normal = \( mg - \frac{mv^2}{r} \)

\[ \text{Normal} = (90\text{kg})(9.8 \frac{\text{m}}{\text{s}^2}) - \frac{(90\text{kg})(15 \frac{\text{m}}{\text{s}})^2}{40\text{m}} \]

Normal = 376N
10. A block of mass \(m = 4\) kg is sitting on a table, pressed against a spring with force constant \(k = 1000\) N/m. Assuming no friction, how far must the spring be compressed so that the mass will fly off the edge of the table at a speed of 5 m/s?

(a) 10 cm  
(b) 22 cm  
(c) 32 cm  
(d) 45 cm

We can use conservation of energy for this one.

Initially, all the energy is stored in the spring. When the spring unloads, the energy is transferred to the block and becomes kinetic energy. There is no friction, so no work done by non-conservative forces. Gravitational potential energy does not change since the table is horizontal, so we can define it to be zero.

\[
K_i + U_i + W_{NC} = K_f + U_f
\]

\[
0 + \frac{1}{2}kx^2 + 0 = \frac{1}{2}mv^2 + 0
\]

\[
\frac{1}{2}(1000\frac{N}{m})x^2 = \frac{1}{2}(4\text{kg})(5\frac{m}{s})^2
\]

\[
x = 0.32m = 32cm
\]
11. A 20 kg child descends a slide that is 4m high and makes an angle of 30 degrees with the ground. At the bottom of the slide the child moves with speed 3 m/s. What is the coefficient of kinetic friction between the child and the slide?

We can use conservation of energy.

Define coordinates so that gravitational potential energy is 0 at the bottom, so the initial energy is mgh.

The final energy is all kinetic.

Kinetic friction does work as the child slides.

\[ K_i + U_i + W_{NC} = K_f + U_f \]
\[ 0 + mgh + W_{friction} = \frac{1}{2}mv^2 + 0 \]
\[ mgh - \mu_k mg \cos(\theta) d = \frac{1}{2}mv^2 \]
\[ \mu_k mg \cos(\theta) \frac{h}{\sin(\theta)} = mgh - \frac{1}{2}mv^2 \]
\[ \mu_k = \frac{(gh - \frac{1}{2}v^2) \sin(\theta)}{gh \cos(\theta)} = 0.51 \]

\[ W_{friction} = -\mu_k mg \cos(\theta) d \]
\[ \sin(\theta) = \frac{h}{d} \rightarrow d = \frac{h}{\sin(\theta)} \]
12. After starting from rest at a height of 40m, a rollercoaster plummets down a steep drop and then completes a loop. Neglecting friction, find the speed of the rollercoaster when it is 20 meters below its starting point. If the mass of the rollercoaster is 2000 kg, what distance is required to stop if a constant braking force of 8000 Newtons is applied after the loop?

We can use conservation of energy for both parts of this problem. Ignoring friction, we are losing potential, and gaining kinetic energy as we fall.

\[ K_i + U_i + W_{NC} = K_f + U_f \]

\[ 0 + mgh + 0 = \frac{1}{2}mv^2 + mg(h - 20m) \]

\[ \frac{1}{2}mv^2 = mg(20m) \]

\[ v^2 = 2g(20m) \rightarrow v = 19.8m/s \]

For the second part, we are ending at ground-level, so define potential energy to be zero there. So we start with potential energy, and end with zero energy (the brakes did work against the motion so KE=0 at the end).

\[ K_i + U_i + W_{NC} = K_f + U_f \]

\[ 0 + mgh + W_{brakes} = 0 + 0 \rightarrow x = \frac{(2000kg)(9.8\frac{m}{s^2})(40m)}{8000 N} \rightarrow x = 98m \]