Practice Midterm #1 Solutions

Physics 6A
1. You drive your car at a speed of 40 km/hr for 1 hour, then slow down to 30 km/hr for the next 20 km. How far did you drive, and what was your average speed?

We can draw a simple diagram with distances and times. For each section we use the formula “Distance = Speed x Time”

Section 1

\[ x_1 = \left(40 \frac{\text{km}}{\text{hr}} \right) (1 \text{hr}) = 40 \text{km} \]

Section 2

\[ 20 \text{km} = \left(30 \frac{\text{km}}{\text{hr}} \right) t_2 \Rightarrow t_2 = \frac{2}{3} \text{hr} \]

The total distance is 60 km.
Total time is 5/3 hr.
Divide these to get the average speed:

\[ \text{avg. speed} = \frac{60 \text{ km}}{\frac{5}{3} \text{ hr}} = 36 \frac{\text{km}}{\text{hr}} \]

answer (b)
2. A speeding car is traveling at 40 m/s down a straight road. The driver slams on the brakes, and the car comes to a complete stop in 5 seconds. Assuming constant acceleration, how far did the car travel while braking?

We can use one of our kinematics formulas.
\[ v^2 = v_0^2 + 2a(x - x_0) \]
This formula will work, but we need to find the acceleration first.

We can use
\[ v = v_0 + at \]
\[ 0 = 40 \text{ m/s} + (a)(5\text{ s}) \]
\[ a = -8 \text{ m/s}^2 \]

Now plug into our previous equation:
\[ v^2 = v_0^2 + 2a(x - x_0) \]
\[ 0^2 = (40 \text{ m/s})^2 + 2(-8 \text{ m/s}^2)(x - x_0) \]
\[ x - x_0 = 100\text{ m} \]

Answer (c)
3. A rock is thrown upward from the top of a 300-meter high cliff. The initial speed is 10 m/s. If we ignore air resistance, how long does it take for the rock to hit the ground below?

First we can draw a sketch and choose a coordinate system. I have chosen the ground as \( y = 0 \). That makes the initial height \( y_0 = 300 \text{m} \).

We also know the initial velocity, and the final height. We can use one of our basic kinematic equations:

\[
y = y_0 + v_0 \cdot t - \frac{1}{2} g \cdot t^2
\]

\[
0 = 300 \text{m} + \left(10 \text{ m/s}\right) \cdot t - \frac{1}{2} \left(9.8 \text{ m/s}^2\right) \cdot t^2
\]

\[
4.9 \cdot t^2 - 10 \cdot t - 300 = 0
\]

\[
t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
t = \frac{-10 \pm \sqrt{10^2 - 4(4.9)(-300)}}{2(4.9)}
\]

\[
t = 8.9 \text{ sec} \quad \text{or} \quad t = -6.9 \text{ sec}
\]

Choose the positive value. Answer (b).
Two rocks are thrown with the same initial speed from the top of a 300-meter high cliff. Rock A is thrown upward and Rock B is thrown downward. If we ignore air resistance, which of the following is true about the speeds of the rocks just before they hit the ground?

a) Rock A is faster.
b) Rock B is faster.
c) Rock A and Rock B have equal speeds.
d) There is not enough information to determine the answer.

The rocks will both land at the same speed. Rock A goes up to some high point, then turns around. By the time it passes the starting point on the way down, it is moving at the initial speed, just opposite direction. So it is just like rock B at this point. Of course rock B hits first, but they are both moving the same speed when they hit.

Answer (c)
5. The radius of Venus is about 6000 km. Assuming Venus is spherical, what is its approximate volume? \( V = \frac{4\pi R^3}{3} \) for a sphere of radius \( R \)

This question is all about units. We are given \( R = 6000 \text{ km} \). We can also convert this into m or cm.

\[
\begin{align*}
R &= 6 \cdot 10^3 \text{ km} \\
R &= 6 \cdot 10^6 \text{ m} \\
R &= 6 \cdot 10^8 \text{ cm}
\end{align*}
\]

Now we can just plug these numbers into our equation:

\[
\begin{align*}
V &= \frac{4\pi}{3} \left(6 \cdot 10^3 \text{ km}\right)^3 = 9.05 \cdot 10^{11} \text{ km}^3 \quad \text{NO} \\
V &= \frac{4\pi}{3} \left(6 \cdot 10^6 \text{ m}\right)^3 = 9.05 \cdot 10^{20} \text{ km}^3 \quad \text{NO} \\
V &= \frac{4\pi}{3} \left(6 \cdot 10^8 \text{ cm}\right)^3 = 9.05 \cdot 10^{26} \text{ cm}^3 \quad \text{YES}
\end{align*}
\]

Answer (d)
6. An airplane is flying East at a speed of 300 km/hr. A gust of wind blows in a Northwest direction at 50 km/hr. Find the new speed and direction of the plane.

We need to add the vectors together, so first we must find their components:

\[
\begin{align*}
  v_{\text{plane}, x} &= 300 \text{ km/hr} \\
  v_{\text{plane}, y} &= 0 \text{ km/hr} \\
  v_{\text{wind}, x} &= -50 \cos(45^\circ) = -35.4 \text{ km/hr} \\
  v_{\text{wind}, y} &= 50 \sin(45^\circ) = 35.4 \text{ km/hr}
\end{align*}
\]

Now add the components together, \( x \) with \( x \) and \( y \) with \( y \).

\[
\begin{align*}
  v_{\text{total}, x} &= 300 - 35.4 = 264.6 \text{ km/hr} \\
  v_{\text{total}, y} &= 35.4 \text{ km/hr}
\end{align*}
\]

Finally, we can put these components together using the Pythagorean Theorem:

\[
|v_{\text{total}}| = \sqrt{(264.6)^2 + (35.4)^2} = 267 \text{ km/hr}
\]

We can calculate the angle from our right-triangle rules, and look at our picture to see that the plane is flying a little bit North of East.

\[
\tan \theta = \frac{35.4}{264.6} \Rightarrow \theta = 7.6^\circ
\]

Answer (c)
7. Bob has 2 pet rocks. Their names are Elvis and Pedro. Bob drops Pedro from a high bridge to the river below. When Pedro has fallen 4m, Bob drops Elvis. As the rocks continue their free-fall, what happens to their separation distance?
   a) The distance increases.
   b) The distance decreases.
   c) The distance stays the same.
   d) There is not enough information to determine the answer.

Think about the speeds of the rocks. Since Pedro gets a head start, he is already moving when Elvis starts to fall. So Pedro is moving faster than Elvis, and they both continue to accelerate at the same rate (gravity). Both speeds keep increasing, but Pedro is always faster. Thus Pedro moves farther than Elvis during each second of their fall. So as they fall, the rocks get farther apart.

Answer (a)
8. A ball is dropped from a train moving on a straight level track at a speed of 50 m/s. The ball is initially 5m above the ground. If air resistance can be ignored, how far does the ball travel horizontally before it hits the ground?

First, draw a picture.

This is a projectile problem, so we use

\[ x = x_0 + v_{0,x}t \]
\[ y = y_0 + v_{0,y}t - \frac{1}{2}gt^2 \]
\[ v_x = v_{0,x} \]
\[ v = v_{0,y} - gt \]
\[ a_x = 0 \]
\[ v^2 = v_{0,y}^2 - 2g(y - y_0) \]

We are given the initial speed (horizontal launch), and the initial height. Notice that to find the horizontal distance, we will need to find the time it takes for the ball to hit the ground. We need to use the first vertical equation:

\[ y = y_0 + v_0 t - \frac{1}{2}gt^2 \]

\[ 0 = 5m + 0 - \frac{1}{2}(9.8 \frac{m}{s^2})t^2 \Rightarrow t = 1.01s \]

Now put this time into the x equation:

\[ x = x_0 + v_{0,x}t \]
\[ x = 0 + (50 \frac{m}{s})(1.01s) = 50.5m \]

Answer (c)
9. A soccer ball is kicked at an angle of $30^\circ$ above the horizontal on a level field. The ball lands 45m from where it was kicked. What was the initial speed of the ball?

Again, start with a picture. The initial velocity will break down into components as follows:

$$v_{0,x} = v_0 \cdot \cos \theta$$
$$v_{0,y} = v_0 \cdot \sin \theta$$

This is a projectile problem, so we use our projectile formulas:

$$x = x_0 + v_{0,x}t$$
$$y = y_0 + v_{0,y}t - \frac{1}{2}gt^2$$
$$v_x = v_{0,x}$$
$$v = v_{0,y} - gt$$
$$a_x = 0$$
$$v^2 = v_{0,y}^2 - 2g(y - y_0)$$

Similar to the previous problem, we can find the time from the $y$ equation, then plug into the $x$-equation.

$$y = y_0 + v_{0,y}t - \frac{1}{2}gt^2$$
$$0 = v_{0,y}t - \frac{1}{2}gt^2$$
$$\frac{1}{2}gt^2 = (v_0 \cdot \sin \theta) \cdot t$$
$$t = \frac{2v_0 \cdot \sin \theta}{g}$$

$$x = x_0 + v_{0,x}t$$
$$45m = 0 + v_0 \cdot \cos \theta \cdot \frac{2v_0 \cdot \sin \theta}{g}$$
$$45m \cdot \left(9.8 \frac{m}{s^2}\right) = 2v_0^2 \cdot \sin(30^\circ) \cdot \cos(30^\circ)$$

$$v_0^2 = \frac{(45m) \cdot \left(9.8 \frac{m}{s^2}\right)}{2 \cdot \sin(30^\circ) \cdot \cos(30^\circ)} \Rightarrow v_0 = 22.6 \frac{m}{s}$$

Answer (c)
10. A golf ball is launched at an angle with an initial speed of 30 m/s, at an angle of 50° above the horizontal. The ball lands on a green that is 5m above where the ball was struck. How far has the ball traveled in the horizontal direction by the time it lands?

First draw a picture, then break the initial velocity into components:

\[ v_{0,x} = (30 \text{ m/s}) \cos(50°) = 19.28 \text{ m/s} \]
\[ v_{0,y} = (30 \text{ m/s}) \sin(50°) = 22.98 \text{ m/s} \]

Again we will use projectile equations:

\[ x = x_0 + v_{0,x} t \]
\[ y = y_0 + v_{0,y} t - \frac{1}{2} g t^2 \]
\[ v_x = v_{0,x} \]
\[ v = v_{0,y} - gt \]
\[ a_x = 0 \]
\[ v^2 = v_{0,y}^2 - 2g(y - y_0) \]

Using \( y_0 = 0 \) and \( y = 5 \text{ m} \) in the first vertical equation, we can find the time:

\[ y = y_0 + v_{0,y} t - \frac{1}{2} g t^2 \]

\[ 5 \text{ m} = 0 + (22.98 \text{ m/s}) t - \frac{1}{2} (9.8 \text{ m/s}^2) t^2 \]

\[ 4.9t^2 - 22.98t + 5 = 0 \]

\[ t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ t = 4.46 \text{ s} \quad \text{or} \quad t = 0.23 \text{ s} \]

The ball is at height 5m twice – once on the way up, and then again on the way down. We want the 4.46s time.

\[ x = x_0 + v_{0,x} t \]
\[ x = 0 + (19.28 \text{ m/s}) \cdot (4.46 \text{ s}) = 86 \text{ m} \]

Answer (a)
11. A golf ball is launched at an angle with an initial speed of 30 m/s, at an angle of 50° above the horizontal. The ball lands on a green that is 5m above where the ball was struck. Find the velocity of the ball just before it lands.

This problem can be done very quickly with no calculations whatsoever if you think about the situation and look at the possible answers given.

You know the speed will be slower than the initial speed because the ball has not yet returned to the initial height. Plus it is on the way down.

The only answer that fits is answer (b).

To calculate the answer, first break the initial velocity into components:

\[ v_{0,x} = (30 \text{ m/s}) \cdot \cos(50°) = 19.28 \text{ m/s} \]
\[ v_{0,y} = (30 \text{ m/s}) \cdot \sin(50°) = 22.98 \text{ m/s} \]

We need to find the components of the final velocity.

We are ignoring air resistance, so we can assume that the x-component of the velocity does not change.

For the y-component, we can use the 3rd of our projectile equations:

\[ v_y^2 = v_{0,y}^2 - 2g(y - y_0) \]
\[ v_y^2 = (22.98 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)(5\text{m} - 0) \Rightarrow v_y = 20.74 \text{ m/s} \]

The magnitude of the velocity is found via the Pythagorean Theorem:

\[ |\vec{v}_{\text{final}}| = \sqrt{(19.28)^2 + (20.74)^2} = 28.32 \text{ m/s} \]

From the diagram, or just common sense, the angle must be below the horizontal (it is on the way down). If you want to do the angle calculation, just use the same tangent formula from previous problems.

Answer (b)
A jet plane comes in for a downward dive as shown in Figure 3.39. The bottom part of the path is a quarter circle having a radius of curvature of 350m. According to medical tests, pilots lose consciousness at an acceleration of 5.5g. At what speed will the pilot black out for this dive?

The given acceleration is 5.5g. This means 5.5 times the acceleration due to gravity:

\[ 5.5 \left( 9.8 \frac{\text{m}}{\text{s}^2} \right) = 53.9 \frac{\text{m}}{\text{s}^2} \]

The path of the airplane is circular, so the given acceleration must be centripetal (toward the center).

We have a formula for centripetal acceleration:

\[ a_{\text{cent}} = \frac{v^2}{r} \]

\[ 53.9 \frac{\text{m}}{\text{s}^2} = \frac{v^2}{350\text{m}} \Rightarrow v = 137 \frac{\text{m}}{\text{s}} \]

Answer (a)
13. Two children are on a merry-go-round which spins at a rate of 1 revolution every 5 seconds. Bobby is standing at a position that is 0.75m from the center, and Cindy is 1.50m from the center. Which statement is true about their linear speeds?

a) Their speeds are equal
b) Bobby’s speed is twice Cindy’s speed.
c) Cindy’s speed is one-and-a-half times Bobby’s speed.
d) Cindy’s speed is twice Bobby’s speed.

We don’t need to do a lot of calculations for this one. Enough information is given to find the speed of each child, but that is too much effort. Understanding the relationship between angular speed and linear speed will make this problem seem easy.

Both children are on the same rotating object, so they must have the same **angular** speed (they rotate at the same rate). However, Cindy is farther from the center, so she has to travel a larger distance every time she goes around. That is, her **linear** speed must be greater. Since she is twice as far from the center, she has to travel twice as far for each rotation, thus her speed is twice as fast.

Answer (d)

Here is the plug-into-the-equations solution (you need to know the circumference of a circle is $2\pi r$):

$$v = \frac{\text{distance}}{\text{time}}$$

$$v_{\text{Bobby}} = \frac{2\pi \cdot (0.75\text{m})}{5\text{s}} = 0.94 \text{ m/s}$$

$$v_{\text{Cindy}} = \frac{2\pi \cdot (1.5\text{m})}{5\text{s}} = 1.88 \text{ m/s}$$