Problem 1

\[ B = \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix} \right\} \] is a basis for \( \mathbb{R}^3 \).

(a) Find the vector \( \vec{x} \) determined by \( [\vec{x}]_B = \begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix} \).

(b) Find the coordinate vector \( [\vec{x}]_B \) of \( \vec{x} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} \) relative to basis \( B \).
Problem 2

The set \( B = \{1 - t^2, 2t + t^2, 1 - t + t^2\} \) is a basis for \( \mathbb{P}_2 \).

Find the coordinate vector of \( \vec{p}(t) = 2 + 3t + 2t^2 \) relative to \( B \).
**Problem 3**

Consider the set of polynomials \( H = \{1 + 4t - t^2, 3t - 1, t^2 - t - 2, 1 + t - t^2\} \).

(a) Is the set linearly independent?
(b) Does the set span \( \mathbb{P}_2 \)?
(c) Could \( H \) be a basis for \( \mathbb{P}_2 \)?
Problem 4

(a) Find the dimension of the subspace spanned by the vectors \([-1, 2, -1], [-2, 1, -2], [4, -2, 0]\).

(b) Determine the dimensions of Nul \(A\) and Col \(A\) for the matrix
\[
\begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
0 & 0 & 1 & 6 & 7 \\
0 & 0 & 0 & 1 & 8 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
Problem 5
(a) If the null space of an $8 \times 5$ matrix $A$ is 2-dimensional, what is the dimension of the row space of $A$?

(b) If $A$ is a $7 \times 5$ matrix, what is the largest possible rank of $A$?

(c) If $A$ is a $6 \times 8$ matrix, what is the smallest possible dimension of $\text{Nul } A$?
Problem 6

Find a basis for the eigenspace corresponding to the eigenvalue.

\[ A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & -13 & 1 \end{bmatrix}, \lambda = -2 \]
Problem 7

(a) What are the eigenvalues of matrix $A$?

$$A = \begin{bmatrix} 1 & 2 & 5 & 100 \\ 0 & 4 & 20 & 3 \\ 0 & 0 & \pi & 10 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$

(b) Is zero an eigenvalue of matrix $B$?

$$B = \begin{bmatrix} 1 & 2 & 5 & 100 \\ 0 & 0 & 20 & 3 \\ 0 & 0 & \pi & 10 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$
Problem 8

Find the characteristic polynomial of the matrix \( A = \begin{bmatrix} 5 & -2 & 3 \\ 0 & 1 & 0 \\ 6 & 7 & -2 \end{bmatrix} \).