Problem 1

Let \( \vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 4 \\ 1 \\ -3 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} -1 \\ -4 \\ 3 \end{bmatrix}, A = [\vec{v}_1 \ \vec{v}_2 \ \vec{v}_3], \) and \( \vec{p} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}. \)

(a) Is \( \vec{p} \) in \( \text{Col } A \)?

(b) Is \( \vec{p} \) in \( \text{Nul } A \)?

Note: Col \( A \) means the “column space of \( A \),” which is the same as the image of \( A \).

Nul \( A \) means the “null space of \( A \),” which is the same as the kernel of \( A \).
Problem 2

Let $A = \begin{bmatrix} 2 & 1 & -1 \\ -5 & 3 & 0 \\ 3 & -1 & 4 \end{bmatrix}$. Compute the det $A$ using the cofactor method and row reduction.
Problem 3

Use a determinant to decide if the vectors \[
\begin{bmatrix}
3 \\
-2 \\
1
\end{bmatrix}, \begin{bmatrix}
1 \\
4 \\
1
\end{bmatrix}, \begin{bmatrix}
0 \\
2 \\
5
\end{bmatrix}
\] are linearly independent.
Problem 4
Let \( H \) be the set of all polynomials of degree at most 3, with integers as coefficients. Is \( H \) a subspace of \( \mathbb{P}_3 \)?
Problem 5

Let $V = \left\{ \begin{bmatrix} 3c - a \\ b \\ a - 2b \\ c + d \end{bmatrix} \mid a, b, c, d \in \mathbb{R} \right\}$.

Show that $V$ is a vector space and find a suitable basis for it.
**Problem 6**

For each of the following sets, answer the following questions:

(1) Are the vectors in the set linearly independent?
(2) Do the vectors in the set span $\mathbb{R}^2$?
(3) Is the set a basis for $\mathbb{R}^2$?

Set $S_1 = \{ \begin{bmatrix} 2 \\ -1 \end{bmatrix} \}$

Set $S_2 = \{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix} \}$

Set $S_3 = \{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \}$

Set $S_4 = \{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \}$