Fundamental Theorem of Calculus

Fundamental Theorem of Calculus, Part 1
If $f$ is continuous on $[a, b]$, then

$$\frac{d}{dx} \int_a^x f(t) \, dt = f(x), \quad a \leq x \leq b$$

Here’s a more general form:

If $F(x) = \int_{u(x)}^{v(x)} f(t) \, dt$, then $F'(x) = f(v(x))v'(x) - f(u(x))u'(x)$.

This theorem basically says that taking the derivative of an antiderivative brings the original function back, but you have to take the chain rule into account!

Example 1
Let’s demonstrate why the theorem works.

(1) Evaluate $\int_0^x (1 - \sin t) \, dt$.

(2) Now take the derivative of the result.

(3) What do you notice?
Example 2
Evaluate the following using the Fundamental Theorem of Calculus.

(a) If \( F(x) = \int_{2}^{x} \sin(t^2) \, dt \), then \( F'(x) = \)

(b) If \( F(x) = \int_{2x}^{3x} \sin(t^2) \, dt \), then \( F'(x) = \)

(c) If \( F(x) = \int_{x^4}^{x^5} (2t - 1)^3 \, dt \), then \( F'(x) = \)
Example 3

Let \( F(x) = \int_{3}^{x} \sqrt{t^2 + 7} \, dt \).

Find:
(a) \( F(3) \)

(b) \( F'(3) \)

(c) \( F''(3) \)
Example 4

Let \( f(x) = \int_0^x \frac{t^2 - 9}{1 + \cos^2 t} \, dt \).

At what value of \( x \) does the local maximum of \( f(x) \) occur?