Antiderivatives, Riemann Sums, Definite Integrals

Example 1
Find the most general antiderivative of each function.

(a) \( f(x) = 3x^2 - 2x + \sqrt{x} \)

(b) \( f(x) = \cos(2x) + \sin(3x) \)

(c) \( f(x) = \frac{1}{3x + 4} \)

(d) \( f(x) = 5e^{4x} \)
Example 2
Write each term of the summation and compute the sum:

\[ \sum_{k=1}^{5} (2k^2 - 3) \]
Example 3
Consider the following summation formulas:

\[\sum_{i=1}^{n} c = cn, \quad \sum_{i=1}^{n} i = \frac{n(n + 1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}, \quad \sum_{i=1}^{n} i^3 = \left[\frac{n(n + 1)}{2}\right]^2\]

Using the appropriate summation formulas, express the following sums in terms of \(n\).

(a) \(\sum_{i=1}^{n} (4 - i)\)

(b) \(\sum_{i=1}^{n} (6i^2 + 4i)\)

(c) \(\sum_{i=1}^{n} (4i^3 + 7)\)
Example 4

Estimate the area under the graph of \( f(x) = 16 - x^2 \) from \( x = 0 \) to \( x = 4 \) by using a Riemann Sum with four rectangles and:

(a) Right endpoints \( (R_4) \)
(b) Left endpoints \( (L_4) \)

Finally, find the exact area using the limit definition \( A = \lim_{n \to \infty} R_n \).
**Example 5**

Evaluate the definite integral:

\[ \int_{0}^{4} (16 - x^2) \, dx \]

Based on your answer to Example 4, what do you think this definite integral represents?
Example 6
Evaluate the following definite integrals. What do you notice?

(a) \[ \int_0^\pi \sin(x) \, dx \]

(b) \[ \int_\pi^{2\pi} \sin(x) \, dx \]

(c) \[ \int_0^{2\pi} \sin(x) \, dx \]