Concavity and L'Hôpital's Rule

Consider the graph of \( f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 1 \).

What's so special about the point \( \left(2, \frac{5}{3}\right) \)?

We call this point an **inflection point** because this is where the curve changes **concavity**.

From \(-\infty < x < 2\), we say the curve is **concave down**.

From \(2 < x < \infty\), we say the curve is **concave up**.

Inflection points occur where:

1. \( f''(x) = 0 \)
2. \( f''(x) \) is undefined if that \( x \) value is in the domain of the function.

However, the curve can also change concavity at vertical asymptotes.

The graph of \( f(x) = \frac{x}{x - 1} \) is concave down to the left of the vertical asymptote and concave up to the right of the vertical asymptote.

There are no inflection points in this graph.
Example 1

Show that \((2, \frac{5}{3})\) is an inflection point of the function \(f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 1\).
Example 2
For the function $f(x) = \sin x - \cos x + 1$, find any inflection points and determine all intervals of concavity on the interval $[0, 2\pi]$. 
Example 3

For \( f(x) = \frac{4x + 1}{2x - 3} \), find any inflection points and determine all intervals of concavity.
Example 4

For $f(x) = \frac{x}{x^2 + x - 6}$, find any inflection points and determine all intervals of concavity.