Properties of Determinants

Example 1
What should be the value of $k$ to make the equation true?

\[
\begin{vmatrix}
4 & -2 & 3 \\
9 & 1 & 7 \\
-5 & 8 & 0
\end{vmatrix}
= k
\begin{vmatrix}
18 & 2 & 14 \\
4 & -2 & 3 \\
-5 & 8 & 0
\end{vmatrix}
\]
Example 2

(a) Find the determinant of \( A = \begin{bmatrix} 1 & 4 & 2 & 3 \\ \pi & -9 & 2\pi & -99 \\ -3 & \ln 7 & -6 & 8 \\ 2 & 100 & 4 & 52 \end{bmatrix} \)

(b) Find the determinant of \( A = \begin{bmatrix} 4 & 10 & -7 & 222 \\ 0 & \pi & 5 & -67 \\ 0 & 0 & e & 59 \\ 0 & 0 & 0 & 3 \end{bmatrix} \).
Example 3

If \[
\begin{bmatrix}
    a & b & c \\
    d & e & f \\
    g & h & i
\end{bmatrix}
\]
= \(-2\), then \[
\begin{bmatrix}
    3g + a & 3h + b & 3i + c \\
    g & h & i \\
    2d & 2e & 2f
\end{bmatrix}
\]
Example 4
If a $4 \times 4$ matrix $A$ with rows $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ has determinant $\det A = 4$, then

$$\det \begin{bmatrix} 2\vec{v}_1 + 3\vec{v}_4 \\ \vec{v}_2 \\ \vec{v}_3 \\ 3\vec{v}_1 + 8\vec{v}_4 \end{bmatrix} = \text{________}.$$
**Example 5**

Consider the following general matrix equation:

\[
\begin{bmatrix}
 a_1 \\
 a_2 \\
\end{bmatrix}
 =
\begin{bmatrix}
 m_{11} & m_{12} \\
 m_{21} & m_{22} \\
\end{bmatrix}
\begin{bmatrix}
 x_1 \\
 x_2 \\
\end{bmatrix}
\]

which can also be abbreviated as:

\[ A = MX \]

By definition, the determinant of \( M \) is given by

\[ \det(M) = m_{11}m_{22} - m_{12}m_{21} \]

The following questions are about the relationship between the determinant of \( M \) and the ability to solve the equation above for \( A \) in terms of \( X \) or for \( X \) in terms of \( A \).

Check the boxes which make the statement correct:

**If the \( \det(M) \neq 0 \) then**

- [ ] A. some values of \( X \) will have no values of \( A \) which satisfy the equation.
- [ ] B. given any \( A \) there is one and only one \( X \) which will satisfy the equation.
- [ ] C. some values of \( A \) (such as \( A = 0 \)) will allow more than one \( X \) to satisfy the equation.
- [ ] D. given any \( X \) there is one and only one \( A \) which will satisfy the equation.
- [ ] E. some values of \( A \) will have no values of \( X \) which will satisfy the equation.
- [ ] F. some values of \( X \) will have more than one value of \( A \) which satisfy the equation.

Check the boxes which make the statement correct:

**If the \( \det(M) = 0 \) then**

- [ ] A. given any \( X \) there is one and only one \( A \) which will satisfy the equation.
- [ ] B. some values of \( A \) will have no values of \( X \) which will satisfy the equation.
- [ ] C. there is no value of \( X \) which satisfies the equation when \( A = 0 \).
- [ ] D. given any \( A \) there is one and only one \( X \) which will satisfy the equation.
- [ ] E. some values of \( A \) (such as \( A = 0 \)) will allow more than one \( X \) to satisfy the equation.

**Check the conditions that guarantee that \( \det(M) = 0 \):**

- [ ] A. When \( A = 0 \) there is more than one \( X \) which satisfies the equation.
- [ ] B. Given any \( A \) the is one and only one \( X \) which will satisfy the equation.
- [ ] C. Given any \( X \) there is one and only one \( A \) which will satisfy the equation.
- [ ] D. There is some value of \( A \) for which no value of \( X \) satisfies the equation.