Properties involving Matrix Inverses and Multiplication

Example 1
If $A$ and $B$ are $3 \times 4$ matrices, and $C$ is a $5 \times 3$ matrix, which of the following are defined?

1. $A^T$

2. $B^T C^T$

3. $C - B$

4. $AB$

5. $CA$

6. $A + B$
Example 2

Find the inverse of $AB$ if $A^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & -4 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$. 
Example 3
If $A$ and $B$ are invertible $n \times n$ matrices, which of the statements below are true?

1. $A + B$ is invertible.

2. $(I_n + A)(I_n + A^{-1}) = 2I_n + A + A^{-1}$

3. $AB = BA$

4. $A^3B^4$ is invertible.

5. $(A + A^{-1})^5 = A^5 + A^{-5}$

Example 4

For what value(s) of $k$ is the matrix $A = \begin{bmatrix} 4 & 1 \\ -3 & k \end{bmatrix}$ invertible?