A function is **separable** if it can be written as the *product* of two functions that have different independent variables:

\[ f(x, y) = h(x)g(y) \]

When a derivative is equal to a separable function, we can separate the functions and integrate both sides to solve the equation.

\[
\frac{dy}{dx} = h(x)g(y) \quad \Rightarrow \quad \int \frac{1}{g(y)}dy = \int h(x)dx
\]

Separable: \[ \frac{dy}{dx} = xy \]  
Not separable: \[ \frac{dy}{dx} = x + y \]

**Example 1**

Find the general solution to the differential equation.

\[
\frac{dy}{dx} = 3y^2 \sin x
\]
**Example 2**
Solve the initial value problem (IVP):
\[
\begin{align*}
\frac{dy}{dt} &= y - 4 \\
y(0) &= -3
\end{align*}
\]
Example 3
Solve the differential equation explicitly for $y$.

\[
\begin{aligned}
\frac{dy}{dt} &= -\frac{x}{y - 3} \\
y(0) &= 1
\end{aligned}
\]
Example 4
Find the general solution to the differential equation $y' = e^{x-y}$. 
Example 5
Find the general solution to the differential equation.
\[ \frac{dy}{dt} = 1 - y^2 \]