**Time Dilation**

- An alien spacecraft is flying overhead at a great distance as you stand in your backyard. You see its searchlight blink on for 0.190 s. The first officer on the craft measures the searchlight to be on for 12.0 ms. (a) Which of these two measured times is the proper time? (b) What is the speed of the spacecraft relative to the earth, expressed as a fraction of the speed of light, \( c \)?

\[ \Delta t = \gamma \Delta t_0 \]

\( \Delta t_0 \) is **Proper Time**, time between events that occur at the same point. So must be measured in a reference frame in which the events are not moving.

The blink occurs at the same point for the person on the spacecraft. Thus the spacecraft is the correct frame of reference to measure proper time of the blink.

\( \Delta t_0 = 12.0 \text{ ms} = 0.0120 \text{s} \).

Since the spacecraft is moving relative to an Earth-based reference frame, the light on event and light off event do not occur at the same point as observed from Earth. Thus \( \Delta t = 0.190 \text{s} \).

\[ \gamma = \frac{\Delta t}{\Delta t_0} = \frac{0.190}{0.0120} = 15.83 \]

\[ \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \quad \Rightarrow \quad \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} \]

\[ \frac{v}{c} = \sqrt{1 - \frac{1}{(15.83)^2}} = \sqrt{0.9960} = 0.998 \]

\[ v = 0.998c \]

\( v \) is the relative velocity of the two reference frames.
Length Contraction

- A meterstick moves past you at great speed. Its motion relative to you is parallel to its long axis. If you measure the length of the moving meterstick to be 1.00 ft (1 ft = 0.3048 m)—for example, by comparing it with a 1-foot ruler that is at rest relative to you, at what speed is the meterstick moving relative to you?

\[ L = \frac{L_0}{\gamma} \]

\( L_0 \) is Proper Length, length of an object measured in a frame of reference for which the object is at rest. \( L \) is measured in a reference frame in which the object is moving. Only the dimension of the object parallel to the direction of relative motion of the two reference frames is affected. Dimensions in directions perpendicular to the relative motion are unaffected.

So, \( L_0 = 1.00 \text{m} \), Proper length you measure the length to be \( L = 0.3048 \) (The meter stick is parallel to the direction of relative motion.)

\[ \gamma = \frac{L_0}{L} = \frac{1.00 \text{m}}{0.3048 \text{m}} = 3.28 \]

\[ \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \rightarrow \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} \]

\[ \frac{v}{c} = \sqrt{1 - \left(\frac{1}{3.28}\right)^2} = 0.952, \quad v = 0.952c \]

\[ v = 0.952 \times 3.00 \times 10^8 \text{m/s} = 2.86 \times 10^8 \text{m/s} \]