MULTIPLE-CHOICE QUESTIONS (20 points). For each of these 5 questions, bubble in the one best answer on your Scantron form AND circle the one best answer on this exam book. Make certain that you do both!

1. Consider the vectors shown. What is the cross product $\vec{C} \times \vec{A}$?

   - A. $+(96.0 \text{ m}) \sin 25.0^\circ \hat{\imath}$
   - B. $+(96.0 \text{ m}) \cos 25.0^\circ \hat{\imath}$
   - C. $-(96.0 \text{ m}) \sin 25.0^\circ \hat{\imath}$
   - D. $-(96.0 \text{ m}) \cos 25.0^\circ \hat{\imath}$
   - E. none of these

   **From right-hand rule, $\vec{C} \times \vec{A}$ points in $+z$-direction**

   $\angle$ between $\vec{C}$ and $\vec{A}$ is $90^\circ - 25.0^\circ$, so magnitude of $\vec{C} \times \vec{A}$ is $(12.0 \text{ m})(8.00 \text{ m}) \sin(90^\circ - 25.0^\circ)$

   $= (96.0 \text{ m}) \cos 25.0^\circ \hat{\imath}$

2. An object is moving along the x-axis. At a certain instant it is speeding up, and the rate at which it is speeding up is increasing. What can you conclude about the x-acceleration $a_x$ of the object at this instant?

   - A. $a_x$ is positive and increasing
   - B. $a_x$ is negative and decreasing (becoming more negative)
   - C. $a_x$ is negative and increasing (becoming less negative)
   - D. either answer A or answer B is possible
   - E. either answer A or answer C is possible

3. The density of a material is equal to its mass divided by its volume. What is the density in kg/m$^3$ of a rock of mass 1.80 kg and volume 6.0 x 10$^{-4}$ m$^3$?

   - A. $3 \times 10^3 \text{ kg/m}^3$
   - B. $3.0 \times 10^3 \text{ kg/m}^3$
   - C. $3.00 \times 10^3 \text{ kg/m}^3$
   - D. $3.000 \times 10^3 \text{ kg/m}^3$
   - E. Any of these — all of these answers are mathematically equivalent.

   **Only B has the correct number of significant figures (see page 8 of your textbook)**

4. Which of the following statements is correct for any two vectors $\vec{A}$ and $\vec{B}$?

   - A. The magnitude of $\vec{A} + \vec{B}$ is greater than or equal to $|\vec{A} - \vec{B}|$
   - B. The magnitude of $\vec{A} + \vec{B}$ is greater than or equal to the magnitude of $\vec{A} - \vec{B}$.
   - C. The magnitude of $\vec{A} + \vec{B}$ is equal to $\sqrt{A^2 + B^2}$.
   - D. More than one of the above statements is correct.
   - E. None of the above statements is correct.

   **The magnitude of $\vec{A} + \vec{B}$ is equal to $|\vec{A} - \vec{B}|$ if $\vec{A}$ & $\vec{B}$ are antiparallel as shown. For any other orientation of the two vectors, the magnitude of $\vec{A} + \vec{B}$ is greater than $|\vec{A} - \vec{B}|$.**

(CONTINUED ON NEXT PAGE)
MUTLIPLE-CHOICE QUESTIONS (continued)

5. The position of an object moving along the x-axis is

\[ x = (2.0 \text{ m/s}^2)t^2 - (6.0 \text{ m/s}^3)t^3 + (3.0 \text{ m/s}^4)t^4 \]

What is the particle doing at \( t = 1.0 \) s?

A. It is moving and speeding up
B. It is moving and slowing down
C. It is moving, but its velocity is not changing at this instant
D. It is momentarily at rest
E. Not enough information given to decide

Velocity of object:

\[ v_x = \frac{dx}{dt} = (2.0 \text{ m/s}^2)(2t) - (6.0 \text{ m/s}^3)(3t^2) + (3.0 \text{ m/s}^4)(4t^3) \]

At \( t = 1.0 \) s,

\[ v_x = (2.0 \text{ m/s}^2)(2)(1.0s) - (6.0 \text{ m/s}^3)(3)(1.0s^2) + (3.0 \text{ m/s}^4)(4)(1.0s^3) \]

\[ = 4.0 \text{ m/s} - 18.0 \text{ m/s} + 12.0 \text{ m/s} = -2.0 \text{ m/s} \]

\( v_x \) is nonzero, so the object is moving.

Acceleration of object:

\[ a_x = \frac{dv_x}{dt} = (2.0 \text{ m/s}^2)(2) - (6.0 \text{ m/s}^3)(6t) + (3.0 \text{ m/s}^4)(12t^2) \]

At \( t = 1.0 \) s,

\[ a_x = (2.0 \text{ m/s}^2)(2) - (6.0 \text{ m/s}^3)(6)(1.0s) + (3.0 \text{ m/s}^4)(12)(1.0s^2) \]

\[ = 4.0 \text{ m/s}^2 - 36.0 \text{ m/s}^2 + 36.0 \text{ m/s}^2 = +4.0 \text{ m/s}^2 \]

\( v_x \) and \( a_x \) have opposite sign, so the object is slowing down.
Problem 1. (This problem is worth 40 points)
A sports car travels in the positive $x$-direction at a constant speed $v_{car}$. At $t = 0$ the sports car drives past a stop sign without stopping, and a policewoman on a motorcycle, which is initially at rest next to the stop sign, begins to move to catch up with the sports car. The policewoman speeds up at a constant rate until $t = T$, when her speed is $(3/2)v_{car}$. She then slows down at a constant rate until $t = 2T$, at which time she is alongside the sports car and traveling at the same speed $v_{car}$ as the sports car.

(a) On the axes below, sketch position-time ($x$-$t$) graphs showing the motion of the sports car and the policewoman. Draw the curves for both the sports car and the policewoman on the same set of axes, and label which curve is which. Be as accurate as possible — accuracy counts!

**Point breakdown for part (a) (SC = sports car, PW = policewoman):**
- 2 points: SC graph shows $x = 0$ at $t = 0$
- 2 points: SC graph is straight line with positive slope
- 2 points: PW graph shows $x = 0$ at $t = 0$
- 2 points: PW graph shows slope $= 0$ at $t = 0$
- 2 points: From $t = 0$ to $t = T$, PW graph is upward parabola
- 2 points: At $t = T$, PW graph has steeper slope than SC graph and has no kinks (slope is continuous)
- 2 points: From $t = T$ to $t = 2T$, PW graph is downward parabola
- 2 points: At $t = 2T$, SC and PW graphs have same value and same slope
- 2 points: PW graph is never above the SC graph

(b) On the axes below, sketch velocity-time ($v_x$-$t$) graphs showing the motion of the sports car and the policewoman. Draw the curves for both the sports car and the policewoman on the same set of axes, and label which curve is which. Be as accurate as possible — accuracy counts!

**Point breakdown for part (b) (SC = sports car, PW = policewoman):**
- 2 points: SC graph is a horizontal line with a positive value
- 2 points: PW graph shows $v_x = 0$ at $t = 0$
- 2 points: From $t = 0$ to $t = T$, PW graph is a straight line with positive slope
- 2 points: At $t = T$, PW graph has a greater value of $v_x$ than SC graph
- 2 points: From $t = T$ to $t = 2T$, PW graph is a straight line with negative slope
- 2 points: At $t = 2T$, SC and PW graphs have the same value of $v_x$
Physics 1 (Freedman) 1st Midterm 1/27/2015

Problem 1 (continued)

(c) Find the distance that the policewoman travels from $t = 0$ to $t = T$. For full credit, show your work, draw a box around your answers, and express your answer in terms of $v_{car}$ and $T$.

\[ \text{(Displacement of policewoman)} = (\text{Policewoman's average velocity}) \times T \]

The policewoman's acceleration over the time interval from $t = 0$ to $t = T$ is constant, so over this interval her average velocity is the average of the velocity at the beginning of the interval (0) and at the end of the interval ($\frac{3}{2} v_{car}$).

So $v_{ave-x} = \frac{1}{2} \left[ 0 + \frac{3}{2} v_{car} \right] = \frac{3}{4} v_{car}$, and her displacement is $\left( \frac{3}{4} v_{car} \right) \times T = \frac{3}{4} \frac{v_{car}}{T}$

(d) Find the distance that the policewoman travels from $t = T$ to $t = 2T$. For full credit, show your work, draw a box around your answers, and express your answer in terms of $v_{car}$ and $T$.

Same as part (c), but now the policewoman's velocity at the beginning of the time interval is $\frac{3}{2} v_{car}$ and at the end of the time interval is $v_{car}$. So $v_{ave-x} = \frac{1}{2} \left[ \frac{3}{2} v_{car} + v_{car} \right] = \frac{1}{2} \left[ \frac{5}{2} v_{car} \right] = \frac{5}{4} v_{car}$, and her displacement is $\left( \frac{5}{4} v_{car} \right) \times T = \frac{5}{4} v_{car} T$

[Note that her total displacement from $t = 0$ to $t = 2T$ is $\frac{3}{4} v_{car} T + \frac{5}{4} v_{car} T = 2v_{car} T$. That's the same as the sports car, which moves at a steady $v_x = v_{car}$ from $t = 0$ to $t = 2T$.]
Problem 2 (This problem is worth 40 points)

A projectile is launched from the edge of the roof of a building as shown. The projectile leaves the roof moving at speed $v_0$ at a 45° angle above the horizontal. The instant before the projectile hits the ground, it is moving at an angle of 53° below the horizontal. Air resistance can be neglected.

Potentially useful information:

\[
\begin{align*}
\sin 45° &= \frac{1}{\sqrt{2}} \\
\cos 45° &= \frac{1}{\sqrt{2}} \\
\tan 45° &= 1 \\
\sin 53° &= \frac{4}{5} \\
\cos 53° &= \frac{3}{5} \\
\tan 53° &= \frac{4}{3}
\end{align*}
\]

(a) How much time elapses from when the projectile is launched to when it hits the ground? For full credit, show your work, simplify your answer, draw a box around your answer, and express your answer in terms of $v_0$ and $g$. Do not insert a numerical value of $g$ — if you do, you will receive zero points for this part.

At $t=0$, projectile has velocity components

\[
\begin{align*}
V_{0x} &= v_0 \cos 45° = \frac{v_0}{\sqrt{2}} \\
V_{0y} &= v_0 \sin 45° = \frac{v_0}{\sqrt{2}}
\end{align*}
\]

When projectile hits the ground, speed is $v$ and velocity components are

\[
\begin{align*}
V_x &= V \cos 53° = \frac{2}{3}V \\
V_y &= -V \sin 53° = -\frac{4}{5}V
\end{align*}
\]

From projectile equs., $V_x = V_{0x} \rightarrow \frac{2}{3}V = \frac{v_0}{\sqrt{2}} \rightarrow V = \frac{5v_0}{3\sqrt{2}}$

and $V_y = V_{0y} - gt \rightarrow gt = V_0y - V_y \rightarrow t = \frac{1}{g}(V_0y - V_y) = \frac{1}{g}(\frac{5v_0}{\sqrt{2}} - (-\frac{4}{5}v_0\frac{2}{3\sqrt{2}})) = \frac{7v_0}{3N2g}$

(b) What distance to the right of the building does the projectile hit the ground? For full credit, show your work, simplify your answer, draw a box around your answer, and express your answer in terms of $v_0$ and $g$. Do not insert a numerical value of $g$ — if you do, you will receive zero points for this part.

From projectile equs.,

\[
\begin{align*}
\Delta x &= \frac{7v_0^2}{6g}
\end{align*}
\]

(Continued on next page)
Problem 2 (continued)

(c) What is the *height* of the building? *For full credit,* show your work, simplify your answer, draw a box around your answer, and express your answer in terms of $v_0$ and $g$. Do not insert a numerical value of $g$ — if you do, you will receive zero points for this part.

\[
y = v_{oy}t - \frac{1}{2}gt^2
\]

\[
-h = \frac{v_0}{\sqrt{2}}t - \frac{1}{2}gt^2
\]

so

\[
h = -\frac{v_0}{\sqrt{2}}t + \frac{1}{2}gt^2 = -\frac{v_0}{\sqrt{2}}\left(\frac{7v_0}{3\sqrt{2}g}\right) + \frac{g}{2}\left(\frac{7v_0}{3\sqrt{2}g}\right)^2
\]

\[
= -\frac{7v_0^2}{6g} + \frac{49v_0^2}{36g} = -\frac{42v_0^2}{36g} + \frac{49v_0^2}{36g} = \frac{7v_0^2}{36g}
\]

(d) *What is the speed of the projectile* just before it hits the ground? *For full credit,* show your work, simplify your answer, draw a box around your answer, and express your answer in terms of $v_0$ and $g$. Do not insert a numerical value of $g$ — if you do, you will receive zero points for this part.

\[
y = v_{oy}t - \frac{1}{2}gt^2
\]

\[
y = v_{oy}t - \frac{1}{2}gt^2
\]

so

\[
v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(\frac{v_0}{\sqrt{2}}\right)^2 + \left(-\frac{7v_0}{3\sqrt{2}}\right)^2}
\]

\[
v = \sqrt{\frac{9v_0^2}{18} + \frac{16v_0^2}{18}} = \sqrt{\frac{25v_0^2}{18}} = \frac{5v_0}{3\sqrt{2}}
\]

Point breakdown for part (c):
- 3 points: using appropriate kinematic equations
- 3 points: using correct vertical component of initial velocity
- 2 points: algebra (including simplifying answer)

*If the student had the wrong value for $t$ from part (a) but used it correctly in part (c), they do not lose any additional points in part (c).*

Point breakdown for part (d):
- 3 points: Using correct horizontal and vertical components of initial velocity
- 3 points: Using correct equations to find components of final velocity
- 2 points: algebra (including simplifying answer)

*If the student had the wrong value for $t$ from part (a) but used it correctly in part (d), they do not lose any additional points in part (d).*