1. (This problem is worth 40 points.) The pulley in the figure has moment of inertia $I$ and radius $R$. It is mounted on frictionless bearings. The lightweight rope does not slip on the pulley rim. Initially the two blocks are at rest. There is friction between the table and the block of mass $m_A$.

(a) Find the *minimum coefficient of static friction* between the table and the block of mass $m_A$ needed to keep the blocks from moving after they are released. Your answer should involve no quantities other than $m_A$, $m_B$, $I$, $R$, and $g$. (It may or may not involve all of these.) For full credit, show your work, simplify your answer, and draw a box around your answer.

![Diagram of pulley system with forces and moments labeled](image)

Equilibrium:

$F_S$ $\rightarrow$ $F_A$ $\rightarrow$ $T$

$\sum F_x = 0$: $T - F_S = 0$

$\sum F_y = 0$: $m_A g - T = 0$

So $F_S = m_B g$ and $F_A = m_A g$

Equilibrium:

$B$ $\rightarrow$ $T$

$\sum F_x = 0$: $T - m_B g = 0$

Equilibrium:

$A$ $\rightarrow$ $F_A$

$\sum F_x = 0$: $F_A - \mu_s F_N = 0$

$\sum F_y = 0$: $m_A g - F_A = 0$

So $F_A = m_A g$ and $F_N = \mu_s m_A g$

Thus $F_S = \mu_s F_N$

Answer must be an equality, not an inequality.

(b) Suppose the coefficient of friction is less than the value that you found in part (a), so the blocks move when they are released. You find that after the block of mass $m_B$ has descended a distance $d$, its speed is $v$. Find the *coefficient of kinetic friction* between the table and the block of mass $m_A$. Your answer should involve no quantities other than $m_A$, $m_B$, $I$, $R$, $g$, $v$, and $d$. (It may or may not involve all of these.) For full credit, show your work, simplify your answer, and draw a box around your answer.

Energy approach:

$K_1 + U_1 + W_{other} = K_2 + U_2$

$K_1 = 0$ $\rightarrow$ $U_1 = m_B g d$

$W_{other} = -F_S d = -\mu_k m_A g d$

$K_2 = \frac{1}{2} m_A v^2 + \frac{1}{2} m_B v^2 + \frac{1}{2} I \omega^2$

$U_2 = 0$

$\omega = \frac{v}{R}$

So $m_B g d = \mu_k m_A g d = \frac{1}{2} m_A v^2 + \frac{1}{2} m_B v^2 + \frac{1}{2} I \left(\frac{v}{R}\right)^2$

Solve for $\mu_k$: $\mu_k = \frac{m_B g d - \frac{1}{2} (m_A + m_B + \frac{I}{R^2}) v^2}{m_A g d}$
2. (This problem is worth 40 points) You are sitting on a sled that is initially at rest on a horizontal sheet of ice. The combined mass of you and the sled is $m_y$. You are holding a bow (mass $m_b$) and arrow (mass $m_a$).

The bow behaves like a spring of force constant $k$. You shoot the arrow at an angle $\theta$ above the horizontal, and it hits the ground a horizontal distance $D$ from where it was fired and at the same elevation from which it was fired. The sled (along with you and the bow) recoils backwards and slides a distance $L$ before coming to a halt.

Find the coefficient of kinetic friction between the ice and the sled. Your answer should involve no quantities other than $m_y$, $m_b$, $m_a$, $k$, $g$, $D$, and $L$. (It may or may not involve all of these.) For full credit, show your work, simplify your answer, and draw a box around your answer.

3 parts:  
- projectile motion - find launch speed of arrow
- momentum - find recoil speed of sled + you + bow
- energy or force & acceleration - find friction force

\[
\text{Projectile motion:} \\
\begin{align*}
x &= D \quad \text{when } y = 0, \quad \text{so } \quad 0 &= (v_0 \cos \theta) t - \frac{1}{2} gt^2 \\
y &= (v_0 \sin \theta) t - \frac{1}{2} gt^2 \\
\end{align*}
\]

Hence
\[
D = (v_0 \cos \theta) \left( \frac{2v_0 \sin \theta}{g} \right) = \frac{2v_0^2 \sin \theta \cos \theta}{g}
\]

\[
V_0 = \sqrt{\frac{gD}{2 \sin \theta \cos \theta}}
\]

\[
\text{Momentum: initial momentum } = 0, \quad \text{so conservation of horizontal momentum says}
\]

\[
0 = m_A V_0 \cos \theta - (m_{ys} + m_B) V_{\text{recoil}}
\]

\[
V_{\text{recoil}} = \frac{m_A}{m_{ys} + m_B} V_0 \cos \theta = \frac{m_A}{m_{ys} + m_B} \sqrt{\frac{gD \cos \theta}{2 \sin \theta}}
\]

\[
\text{Energy (easiest): } W_{\text{tot}} = \Delta K \Rightarrow -m_k (m_{ys} + m_B) gL = -\frac{1}{2} (m_{ys} + m_B) V_{\text{recoil}}^2
\]

\[
\text{so } m_k = \frac{V_{\text{recoil}}^2}{2gL} = \left( \frac{m_A}{m_{ys} + m_B} \right)^2 \cdot \frac{1}{2gL} \cdot \frac{gD \cos \theta}{2 \sin \theta}
\]

\[
= \left( \frac{m_A}{m_{ys} + m_B} \right)^2 \cdot \frac{D \cos \theta}{4L \sin \theta}
\]
3. (This problem is worth 40 points) A bowling ball of mass $m$ is attached to the ceiling by a lightweight rope of length $L$. The ball is pulled to the left until the rope is at a 45° angle with the vertical. The ball is then released from rest. Neglect air resistance.

**Possibly useful information:** $\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$, $\tan 45^\circ = 1$. You can do part (c) without first having done part (b).

(a) In the space below, **draw two free-body diagrams** for the ball after it is released: One diagram for the instant where the rope is vertical, and one diagram for the instant when the ball has swung to the right so that the rope is again at a 45° angle with the vertical. For each force in your diagram, indicate what object exerts that force. In addition, in each diagram **indicate the direction of the ball’s acceleration (if any).**

(b) Find the **tension in the rope** at the instant when the rope is vertical. Your answer should involve no quantities other than $m$, $L$, and $g$. (It may or may not involve all of these.) For full credit, **show your work, simplify your answer, and draw a box around your answer.**

\[
\Sigma F_x = T - mg = ma
\]

\[
a = \frac{v^2}{L}
\]

\[
T = mg + ma
\]

Ball has descended a distance $L(1 - \frac{1}{\sqrt{2}})$

so $\Delta U = -mgL\left(1 - \frac{1}{\sqrt{2}}\right)$

This equals $\Delta K$, so $K$ at bottom is $\frac{1}{2}mv^2 + mgL\left(1 - \frac{1}{\sqrt{2}}\right)$

so $T = mg + \frac{mv^2}{2} = mg + 2K = mg + g\left(2 - \frac{2}{\sqrt{2}}\right) = mg\left(3 - \sqrt{2}\right)$

(b) Find the **tension in the rope** at the instant when the ball has swung to the right so that the rope is again at a 45° angle with the vertical. Your answer should involve no quantities other than $m$, $L$, and $g$. (It may or may not involve all of these.) For full credit, **show your work, simplify your answer, and draw a box around your answer.**

\[
\Sigma F_x = mg \cos 45^\circ = ma
\]

\[
\Sigma F_y = T - mg \sin 45^\circ = 0
\]

so $T = mg \sin 45^\circ = \frac{mg}{\sqrt{2}}$
4. (This problem is worth 40 points) You stand in the door of a cheap motel and turn on the lights, you see a cockroach a distance \( D \) from the door, scurrying directly away from you at a constant speed \( v_c \). Starting from rest, you move toward the cockroach at a constant acceleration and catch it with your foot just as the cockroach ducks under a counter a distance \( L \) from the door.

(a) Using the axes below, draw an \( x-t \) graph and a \( v_t-A \) graph that show the motion of both you and the cockroach. On each graph, label which curve shows your motion and which curve shows the cockroach’s motion. On each graph, label the time when you catch up to the cockroach.

(b) Find your acceleration as you run to catch up with the cockroach. Your answer should involve no quantities other than \( D \), \( L \), and \( v_c \). (It may or may not involve all of these.) For full credit, show your work, simplify your answer, and draw a box around your answer.

\[
x_{\text{you}} = \frac{1}{2} a t^2
\]

\[
x_{\text{cockroach}} = D + v_c t
\]

You meet at \( x = L \) so \( L = \frac{1}{2} a t^2 \) catch = \( D + v_c t \) catch \( \mathbf{+ 3} \)

Hence \( t_{\text{catch}} = \frac{L - D}{v_c} \)

\[a = \frac{2L}{t_{\text{catch}}^2} = \frac{2L \cdot \frac{v_c^2}{(L-D)^2}}{2L} \mathbf{+ 2}\]

(c) Find your speed at the instant that you catch up with the cockroach. Your answer should involve no quantities other than \( D \), \( L \), and \( v_c \). (It may or may not involve all of these.) For full credit, show your work, simplify your answer, and draw a box around your answer.

\[
V = at_{\text{catch}} = \left[\frac{2L \cdot \frac{v_c^2}{(L-D)^2}}{2L} \right] \left[\frac{(L-D)}{v_c}\right] \text{ from (b)}
\]

\[= \frac{2Lv_c}{(L-D)} \mathbf{+ 2}\]
5. MULTIPLE CHOICE. (This problem is worth 40 points.)
For each question, **draw a circle** around the **one best answer**. Each question is worth 4 points. There is no penalty for guessing.

(a) A disk starts from rest and rotates with constant angular acceleration. If the angular velocity is $\omega$ at the end of the first two revolutions, then at the end of the first eight revolutions it will be

(i) $\omega \sqrt{2}$
(ii) $2\omega$
(iii) $4\omega$
(iv) $16\omega$

(b) You wish to accelerate your car from rest at a constant acceleration. Assume that there is negligible air drag. To cause a constant acceleration, the car's engine must

(i) maintain a constant power output
(ii) develop ever-decreasing power
(iii) develop ever-increasing power
(iv) develop increasing power at low speeds and decreasing power at high speeds
(v) develop decreasing power at low speeds and increasing power at high speeds

(c) An artist wearing spiked shoes pushes two crates across her frictionless horizontal studio floor. If she exerts a horizontal 36-N force on the smaller crate, then the smaller crate exerts a force on the larger crate that is closest to

(i) 36 N
(ii) 30 N
(iii) 200 N
(iv) 240 N

(d) A force $F$ is required to push a crate along a rough horizontal floor at a constant speed $v$ with friction present. What force is needed to push this crate along the same floor at a constant speed $3v$ if friction is the same as before?

(i) A constant force $3F$ is needed
(ii) A force that gradually increases from $F$ to $3F$ is needed
(iii) A constant force $3F$ is needed
(iv) No force is needed, since the crate has no acceleration

(CONTINUED ON NEXT PAGE)
5. (continued)

(c) A redwood cone and an acorn fall from rest at the same instant and encounter no appreciable air resistance, but the redwood cone started from a height above the ground that was three times as high as that of the acorn. If the redwood cone takes time $t$ to reach the ground, the time that the acorn takes is

(i) $t/3$
(ii) $t/\sqrt{3}$
(iii) $t\sqrt{3}$
(iv) $3t$

(f) A 3-lb physics book rests on an ordinary scale that is placed on a horizontal table. The reaction force to the downward 3-lb force on the book is

(i) an upward 3-lb force on the scale due to the table
(ii) an upward 3-lb force on the book due to the scale
(iii) an upward 3-lb force on the table due to the floor
(iv) an upward 3-lb force on the earth due to the book

(g) A mass $m$ is on top of a horizontal, frictionless table. When it moves in a horizontal circle with speed $v$ at the end of a string of length $L$, the tension in the string is $T$. If the speed of this mass were doubled, the tension would be

(i) $2T$
(ii) $4T$
(iii) $T\sqrt{2}$
(iv) $\frac{T}{ \sqrt{2} }$

(h) A spiral spring is initially relaxed. It is then compressed so as to add $U$ units of potential energy to it. When this spring is instead stretched two-thirds of the distance it was compressed, its potential energy in the same units will be

(i) $2U/3$
(ii) $4U/9$
(iii) $U/3$
(iv) $U/9$

(CONTINUED ON NEXT PAGE)
5. (continued)

(i) The figure shows the velocity of a jogger as a function of time. Which statement about the jogger’s motion is correct?

(i) Her speed and acceleration are both increasing
(ii) Her speed and acceleration are both decreasing
(iii) Her speed is increasing and her acceleration is decreasing
(iv) Her speed is decreasing and her acceleration is increasing
(v) Her speed is increasing and her acceleration is constant
(vi) Her speed is decreasing and her acceleration is constant

(j) If vector $\vec{A}$ has components $A_x$ and $A_y$ and makes an angle $\theta$ with the +x axis, then

(i) $|\vec{A}| = A_x + A_y$
(ii) $\theta = A_y / A_x$
(iii) $\cos \theta = \frac{A_x}{\sqrt{A_x^2 + A_y^2}}$
(iv) $\tan \theta = A_y / A_x$

END OF THE EXAM