Jumping on a Merry-go-Round

A girl \( m_g = 30 \text{ kg} \) is running at \( 2 \text{ m/s} \) and jumps on a turning merry-go-round (MGR). The MGR (modeled as a solid disc) has \( M = 200 \text{ kg} \) and \( R = 2 \text{ m} \). It turns at \( 1 \text{ rad/s} \) (clockwise, thus \( \omega_{\text{MGR}} = -1 \text{ rad/s} \)). The MGR turns toward the girl. Her velocity path makes an angle of \( 30^\circ \) with respect to a tangent to the MGR. See Figure. She lands at a point \( r = 1.5 \text{ m} \) from the MGR’s axis of rotation. Ignore any friction at the axis of rotation.

\[ \theta = 120^\circ \]

What is the angular velocity of the MGR with girl after she jumps on?

This is an angular collision. Since \( T_{\text{ex}} = 0 \), the angular momentum is conserved. The angular momentum is calculated with respect to the axis of rotation.

\[ \Delta L = 0 \rightarrow L_i = L_f \]

The initial angular momentum is the sum of the MGR’s \( I_{\text{MGR}} \) \( \omega_i \) and the \( I_g \) of the girl.

\[ I_{\text{MGR}} = \frac{1}{2} M R^2 = \frac{1}{2} (200 \text{ kg})(2 \text{ m})^2 = 400 \text{ kg} \cdot \text{m}^2 \]

\[ L_{\text{MGR}} = 400 \text{ kg} \cdot \text{m}^2 (-1 \text{ rad/s}) = -400 \text{ kg} \cdot \text{m}^2 \cdot \text{s} \]

(\(-z\) direction)

The girl has linear momentum of magnitude

\[ p_g = m_g v_g = 30 \text{ kg} \times 2 \text{ m/s} = 60 \text{ kg} \cdot \text{m/s} \]
Her initial angular momentum with respect to the axis of rotation is
\[
\vec{L}_i = \vec{r} \times \vec{p}_g
\]
\[
L_i = r \ p \ \sin \theta
\]
\(\theta\) is the angle between the \(\vec{r}\) direction and her \(\vec{p}_g\) direction. The direction of \(L_i\) is given by the right-hand sign rule.

From the figure we see \(\theta = 120^\circ\).

Thus \(L_i = 1.5 \ m \ (60 \ kg \ m/s) \ \sin 120^\circ = 77.9 \ kg \ m^2/s\)
The direction is counter-clockwise, i.e. positive. (+z direction)

Let's use \(\vec{L}_g = \vec{r} \times \vec{p}_g\) \(\vec{F} = 1.5 \ \hat{i} + 0 \ \hat{j} + 0 \ \hat{k}\)
\(\vec{p}_g = -p \ \sin 30 \ \hat{i} + p \ \cos 30 \ \hat{j} + 0 \ \hat{k}\)
\[\frac{-p \ \sin 30 \ \hat{i} + p \ \cos 30 \ \hat{j} + 0 \ \hat{k}}{= -60 \ \sin 30 \ \hat{i} + 60 \ \cos 30 \ \hat{j} + 0 \ \hat{k}}\]
\(\vec{r} \times \vec{p}_g = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.5 & 0 & 0 \\ -30 & 52 & 0 \end{vmatrix} = 0 \ \hat{i} + 0 \ \hat{j} + 1.5 \times 52 \ \hat{k}\)

in agreement.

\[L_i = -400 \ kg \ m^2/s + 77.9 \ kg \ m^2/s = -322.1 \ kg \ m^2/s\]
\[L_f = L_i = -322.1 \ kg \ m^2/s = I_{f \ \text{Total}} \ \omega_f \ \rightarrow \ \omega_f = \frac{L_f}{I_{\text{Total}}}\]

\[I_{\text{Total}} = I_{\text{Mom}} + I_g , \ I_{\text{Mom}} = 400 \ kg \ m^2 \ (\text{no change})\]

\(I_g\) is the moment of inertia of a point mass
\[I_g = m \ g (r)^2 = 30 \ kg \times (1.5 \ m)^2 = 67.5 \ kg \ m^2\]
\[I_{\text{Total}} = 400 \ kg \ m^2 + 67.5 \ kg \ m^2 = 467.5 \ kg \ m^2\]

Thus \(\omega_f = \frac{L_f}{I_{\text{Total}}} = -\frac{322.1 \ kg \ m^2/s}{467.5 \ kg \ m^2} = -0.689 \ \text{rad/s}\)