Parabola - Sketch and Tangent Line

Given \( y = f(x) = -x^2 + 4x - 1 \)

1. Sketch
2. Given that \( f(x) \) has a slope of 2 when \( x = 1 \), what is the equation of a tangent line to \( f(x) \) when \( x = 1 \)?

\[
1. \quad -x^2 + 4x - 1 = -(x^2 - 4x + 1)
\]

Complete Square
\[
- (x - 2)^2 - 3
\]

\[
y = -(x - 2)^2
\]

\[
y = -x^2
\]

\[
y = -(x - 2)^2 + 3
\]

\[
y = 2x
\]

\[
y = -x^2 + 4x - 1
\]

Let's refine \( y = 0 \) → \( x^2 - 4x + 1 = 0 \)

\[
x = \frac{4 \pm \sqrt{(4)^2 - 4(1)(1)}}{2 \cdot 1} = \frac{2 \pm \sqrt{3}}{1}
\]

\[
x = 2 - \sqrt{3} \approx 0.268 \quad x = 2 + \sqrt{3} \approx 3.73
\]

Where does \( f(x) \) cross y axis, \( @ x = 0 \)

\[
y = f(0) = -1
\]
Now the tangent line. We need the point where tangent line is constructed. We were given \( x = 1 \) need \( y = f(x=1) \).

\[ y = -(1)^2 + 4(1) - 1 = -1 + 4 - 1 = 2 \]

The point is \((1, 2)\).

**Point-Slope Equation of Line**

\[
\frac{y - y_0}{x - x_0} = m \quad m \text{ is slope at point } (x_0, y_0) \text{ of function.}
\]

Thus

\[
\frac{y - 2}{x - 1} = 2
\]

\[ y - 2 = 2(x - 1) = 2x - 2 \]

\[ y = 2x \]

The tangent line has been added to previous graph.