Applying Stokes' Theorem
Let $S$ be the surface $z = 8 - 2x^2 - 2y^2$, z > 0 oriented upward. Let
\[ \mathbf{F} = (x + y\cos z, 2x e^z - y, x^3 y^2 - z^4) \]
Evaluate
\[ \iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} \]
Stokes' Theorem says
\[ \oint_{\partial S} \mathbf{F} \cdot d\mathbf{S} = \iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} \]
Visualize $S$

This is a paraboloid of revolution about the $z$ axis and above the $z = 0$ plane (ie x-y plane)
The boundary curve $\mathbf{\Gamma}$ is a circle in the x-y plane of radius 2.
(Let $z = 0 \rightarrow 2x^2 + 2y^2 = 8 \rightarrow x^2 + y^2 = 4 = 2^2$)
So $\mathbf{\Gamma} = (2 \cos t, 2 \sin t, 0)$, $0 \leq t \leq 2\pi$
This is as illustrated and is the correct direction for $S$ oriented upward (and outward).
\[ \oint_{\mathbf{\Gamma}} \mathbf{F} \cdot d\mathbf{S} = \int^{2\pi}_0 \mathbf{F}(\mathbf{\Gamma}) \cdot \mathbf{\Gamma}' \, dt \]

$\mathbf{F}(\mathbf{\Gamma}) = (2 \cos t + 2 \sin t, 4 \cos t - 2 \sin t, 32 \cos^3 t \sin t)$
(Having set $z = 0$)

$\mathbf{\Gamma}' = (-2 \sin t, 2 \cos t, 0)$
$\mathbf{F} \cdot \mathbf{\Gamma}' = -8 \sin t \cos t - 4 \sin^2 t + 8 \cos^2 t - 4 \sin t \cos t$
Combining some \( \sin^2 \theta \) and \( \cos^2 \theta \), this simplifies to:
\[
-8 \sin \theta \cos \theta + 4 + 12 \cos^2 \theta
\]
So \[
\int \mathbf{F} \cdot d\mathbf{s} = \int_0^{2\pi} (-8 \sin \theta \cos \theta + 4 + 12 \cos^2 \theta) \, d\theta
\]

let \( \cos^2 \theta = \frac{1}{2} + \frac{\cos 2\theta}{2} \) and we have
\[
= \int_0^{2\pi} (-8 \sin \theta \cos \theta + 2 + 6 \cos 2\theta) \, d\theta
\]
\[
= \left[ (-4 \sin^2 \theta + 2t + 3 \sin 2\theta) \right]_0^{2\pi}
\]
\[
= 0 + 4\pi + 0 - (0 + 0 + 0) = 4\pi
\]

This was not too bad, but is there an alternative? The actual surface integral for the given \( \mathbf{F} \) and \( S \) looks really bad. But Stokes' Theorem also says:
\[
\int \mathbf{F} \cdot d\mathbf{s} = \iint (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \iint (\nabla \times \mathbf{F}) \cdot d\mathbf{S}^*
\]
\[
\bar{c} = \partial S \quad S \quad S^*
\]

That is, another surface may be used in taking the surface integral as long as it satisfies the required "niceness" properties, has the same orientation, and shares the same boundary curve, i.e.
\[
\bar{c} = \partial S = \partial S^*
\]
we are free to pick a different surface \( S^* \) which makes the surface integral easier to evaluate!
What could be nicer to work with than a flat disk of radius \( z \) in the \( x-y \) plane for which \( z \) is still the boundary curve?

Parameterize such a disk,

\[
\vec{r}(u,v) = (ucosv, usinv, 0) \quad 0 \leq u \leq 2 \\
0 \leq v \leq 2\pi
\]

\[
\vec{r}_u = (cosv, -usinv, 0) \quad D_{u,v}^x \\
\vec{r}_v = (-usinv, ucosv, 0)
\]

\[
\vec{N} = \vec{r}_u \times \vec{r}_v = \begin{vmatrix}
\cos v & \sin v & k \\
-\sin v & \cos v & 0 \\
-uscinv & usinv & 0
\end{vmatrix} = \hat{c}(0) + \hat{j}(0) + \hat{k}(ucos^2v + usinv^2) = (0, 0, u)
\]

Since the \( z \) component is positive, this disk is oriented upward the same as the original surface. Thus we evaluate

\[
\iint_{\Sigma^*} (\vec{\nabla} \times \vec{F}) \cdot d\Sigma^{*} = \iint_{D_{u,v}^*} (\vec{\nabla} \times \vec{F}(\vec{r})) \cdot \vec{N} \ dudv
\]

Since \( \vec{N} \) has only a \( z \) component, we need only the \( z \) component of \( \vec{\nabla} \times \vec{F}(\vec{r}) \) to take the dot product, i.e.

\[
(\vec{\nabla} \times \vec{F}) \cdot \hat{k} = \left( \frac{\partial F_z}{\partial x} - \frac{\partial F_y}{\partial y} \right)
\]

For \( \vec{F} = (F_1, F_2, F_3) \)

\[
\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = \frac{2}{\partial x}(2xe^{-y}) - \frac{2}{\partial y}(x + y \cos z)
\]

\[= 2e^2 - \cos z\]
But this is evaluated for the $F(u,v)$ associated with $S^*$, for which $z = 0$.
Thus $\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 2e^0 - \cos 0 = 2 - 1 = 1$

Wow! How things have simplified!

$$\iint_{D^*_{uv}} (\nabla \times \overline{F}(F)) \cdot \overline{N} \, du \, dv = \iiint_{D^*_{uv}} u \, du \, dv$$

$$= \int_0^{2\pi} \int_0^u \frac{u^2}{2} \, dv = \int_0^{2\pi} 2v \, dv = 2v \bigg|_0^{2\pi} = 4\pi$$

Since this evaluated surface integral and the desired surface integral are both equal to the same path integral, they are equal to each other.

By using a different surface with the same boundary curve the normal is pointing in only one direction and the curl of $\overline{F}$ greatly simplifies and the integration becomes trivial.