**Torque**

A torque is the twisting or turning effort of a force as it is applied to an object which is not a point mass. A torque is always defined with respect to a specific point, often identified with a \( O \).

The force \( \vec{F} \) is applied at point \( P \) which is located at \( \vec{F} \) from \( O \). Actually, \( P \) only needs to be any point on the line of action of \( \vec{F} \).

The torque is a vector given by taking the vector cross product of \( \vec{F} \) and \( \vec{F} \), i.e.

\[
\vec{T}_O = \vec{r} \times \vec{F}
\]

The magnitude of \( \vec{T}_O \) is \( \vec{T}_O = r F \sin \phi \), where \( r \) and \( F \) are the magnitudes of vectors \( \vec{r} \) and \( \vec{F} \) and \( \phi \) is the angle between the direction of \( \vec{r} \) and the direction of \( \vec{F} \).

We are mostly concerned with \( \vec{F} \) and \( \vec{F} \) in the \( x-y \) plane. Then \( \vec{T} \) is either in the \( +z \) or the \( -z \) direction only. For the illustrated case, with a right handed coordinate system, the torque is counter clockwise (ccw) and thus in the \( +z \) direction.

\[
\tau = Fl = F_{\text{tan}} r = Fr \sin \phi
\]

\[
l = r \sin \phi = \text{lever arm}
\]

10.3 The torque of the force \( \vec{F} \) about the point \( O \) is defined by \( \tau = \vec{r} \times \vec{F} \). The magnitude of \( \tau \) is \( Fr \sin \phi \). In this figure, \( \vec{r} \) and \( \vec{F} \) are in the plane of the page and \( \vec{r} \) points out of the page toward you.
However, in particular problems when the vector cross product is not explicitly used we can take clockwise torques as being positive as long as we are consistent. We can view $T_o$ as

$$T_o = (r \sin \phi) \mathbf{F} = \ell \mathbf{F}, \quad \ell = r \sin \phi$$

or as

$$T_o = r (F \sin \phi) = r F_{\tan}, \quad F_{\tan} = F \sin \phi$$

where $\ell$ is called the "lever arm" and is the perpendicular distance from $O$ to the force's line of action, and $F_{\tan}$ is the tangential component of $\mathbf{F}$ and is perpendicular to $\mathbf{F}$.

Let's do an example in the $x$-$y$ plane.

\[ F \text{ is applied at } P, \text{ located at } F=(4,2,0). \text{ But as a vector based at the origin, } F=(1,3,0) \]

\[
\begin{align*}
\mathbf{T}_o &= \mathbf{r} \times \mathbf{F} = (4,2,0) \times (1,3,0) \\
&= \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
4 & 2 & 0 \\
1 & 3 & 0 \\
\end{vmatrix} = \mathbf{i}(0) + \mathbf{j}(0) + \mathbf{k}(12 - 2) \\
&= (0, 0, 10) \quad T_o = 10 \text{ N} \cdot \text{m} \ \text{o}
\end{align*}
\]

\[
\mathbf{r} = \sqrt{16 + 4} = \sqrt{20} \quad F = \sqrt{1 + 9} = \sqrt{10}
\]
We can use the vector dot product to find angle $\phi$.

$$\phi = \cos^{-1} \left( \frac{\vec{F} \cdot \vec{F}}{rF} \right) = \cos^{-1} \left( \frac{(4,2,0) \cdot (1,3,0)}{\sqrt{20} \sqrt{10}} \right)$$

$$= \cos^{-1} \left( \frac{10}{\sqrt{200}} \right) = \cos^{-1}(0.7071) = 45^\circ$$

Thus $l = r \sin \phi = \sqrt{20} \sin 45^\circ = 3.162$

$$F_{\text{tan}} = F \sin \phi = \sqrt{10} \sin 45^\circ = 2.236$$

$$T_0 = lF = 3.162 \times \sqrt{10} = 10$$

$$T_0 = rF_{\text{tan}} = \sqrt{20} \times 2.236 = 10$$

Do this problem

10.3 A square metal plate 0.180 m on each side is pivoted about an axis through point $O$ at its center and perpendicular to the plate (Fig. 10.40). Calculate the net torque about this axis due to the three forces shown in the figure if the magnitudes of the forces are $F_1 = 18.0$ N, $F_2 = 26.0$ N, and $F_3 = 14.0$ N. The plate and all forces are in the plane of the page.

I suggest using the idea $T_0 = lF$ and using the components of $F_3$.

Answer: 2.44 N-m

At what angle would $F_3$ produce the largest torque?