9.84 Exactly one turn of a flexible rope with mass $m$ is wrapped around a uniform cylinder with mass $M$ and radius $R$. The cylinder rotates without friction about a horizontal axle along the cylinder axis. One end of the rope is attached to the cylinder. The cylinder starts with an angular speed $\omega_0$. After one revolution of the cylinder the rope has unwrapped and, at this instant, hangs vertically down, tangent to the cylinder. Find the angular speed of the cylinder and the linear speed of the lower end of the rope at this time. You can ignore the thickness of the rope. (Hint: Use Eq. (9.18).)

**IDENTIFY:** Apply conservation of energy to the system of cylinder and rope.

**SET UP:** Taking the zero of gravitational potential energy to be at the axle, the initial potential energy is zero (the rope is wrapped in a circle with center on the axle). When the rope has unwound, its center of mass is a distance $\pi R$ below the axle, since the length of the rope is $2\pi R$ and half this distance is the position of the center of the mass. Initially, every part of the rope is moving with speed $a_0 R$, and when the rope has unwound, and the cylinder has angular speed $\omega$, the speed of the rope is $\omega R$ (the upper end of the rope has the same tangential speed at the edge of the cylinder). $I = (1/2)MR^2$ for a uniform cylinder.

**EXECUTE:**

\[ K_1 = K_2 + U_2, \quad \left( \frac{m}{4} + \frac{m}{2} \right) R^2 a_0^2 = \left( \frac{M}{4} + \frac{m}{2} \right) R^2 \omega^2 - mgR. \]

Solving for $\omega$ gives

\[ \omega = \sqrt{\frac{a_0^2 + \frac{4\pi mg}{R}}{2m}}, \text{ and the speed of any part of the rope is } v = \omega R. \]

**EVALUATE:** When $m \to 0$, $\omega \to \omega_0$. When $m \gg M$, $\omega = \sqrt{\frac{a_0^2 + \frac{2\pi g}{R}}{R}}$ and $v = \sqrt{v_0^2 + 2\pi g R}$. This is the final speed when an object with initial speed $v_0$ descends a distance $\pi R$. 

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**Diagram:**

- **Initial 1**
  - $\omega_1 = \omega_0$
  - c.m. of Both rope and disc

- **Final 2**
  - $\omega_2$
  - End of Rope

- $2\pi R$
  - Rope C.m.
  - End of Rope
9.85 The pulley in Fig. 9.30 has radius $R$ and a moment of inertia $I$. The rope does not slip over the pulley, and the pulley spins on a frictionless axle. The coefficient of kinetic friction between block $A$ and the tabletop is $\mu_k$. The system is released from rest, and block $B$ descends. Block $A$ has mass $m_A$ and block $B$ has mass $m_B$. Use energy methods to calculate the speed of block $B$ as a function of the distance $d$ that it has descended.

![Pulley Diagram](image)

**Figure 9.30** Problem 9.85.

**IDENTIFY:** Apply conservation of energy to the system consisting of blocks $A$ and $B$ and the pulley.

**SET UP:** The system at points 1 and 2 of its motion is sketched in Figure 9.83.

![Energy Diagram](image)

**Figure 9.83**

Use the work-energy relation $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Use coordinates where $+y$ is upward and where the origin is at the position of block $B$ after it has descended. The tension in the rope does positive work on block $A$ and negative work of the same magnitude on block $B$, so the net work done by the tension in the rope is zero. Both blocks have the same speed.

**EXECUTE:** Gravity does work on block $B$ and kinetic friction does work on block $A$. Therefore

$W_{\text{other}} = W_T = -\mu_k m_A gd$.

$K_1 = 0$ (system is released from rest)

$U_1 = m_B g y_1 = m_B gd; \quad U_2 = m_B g y_2 = 0$

$K_2 = \frac{1}{2} m_A v^2_A + \frac{1}{2} m_B v_B^2 + \frac{1}{2} I \omega_2^2$.

But $v$(blocks) = $R\omega$(pulley), so $\omega_2 = v_2/R$ and

$K_2 = \frac{1}{2} (m_A + m_B) v_2^2 + \frac{1}{2} I (v_2/R)^2 = \frac{1}{2} (m_A + m_B + I/R^2) v_2^2$

Putting all this into the work-energy relation gives

$m_B gd - \mu_k m_A gd = \frac{1}{2} (m_A + m_B + I/R^2) v_2^2$

$(m_A + m_B + I/R^2) v_2^2 = 2 gd (m_B - \mu_k m_A)$

$v_2 = \sqrt{\frac{2gd (m_B - \mu_k m_A)}{m_A + m_B + I/R^2}}$

**EVALUATE:** If $m_B > m_A$ and $I/R^2$, then $v_2 = \sqrt{2gd}$; block $B$ falls freely. If $I$ is very large, $v_2$ is very small. Must have $m_B > \mu_k m_A$ for motion, so the weight of $B$ will be larger than the friction force on $A$.

$I/R^2$ has units of mass and is in a sense the "effective mass" of the pulley.