The earth, which is not a uniform sphere, has a moment of inertia of 0.3308MR\(^2\) about an axis through its north and south poles. It takes the earth 86,164 s to spin once about this axis. Use Appendix F to calculate a) the earth’s kinetic energy due to its rotation about this axis and b) the earth’s kinetic energy due to its orbital motion around the sun. c) Explain how the value of the earth’s moment of inertia tells us that the mass of the earth is concentrated toward the planet’s center.

IDENTIFY: \(K = \frac{1}{2} I \omega^2\), where \(I\) is the period of the motion. For the earth’s orbital motion it can be treated as a point mass and \(I = MR^2\).

SET UP: The earth’s rotational period is 24 h = 86,164 s. Its orbital period is 1 yr = 3.156\(\times10^7\) s.

\(M = 5.97\times10^{24}\) kg, \(R = 6.38\times10^6\) m.

EXECUTE: (a) \(K = \frac{2\pi^2 I}{7^2} = \frac{2\pi^2 (0.3308)(5.97\times10^{24}\) kg/(6.38\times10^6\) m\(^2\)}{(86.164\) s\(^2\)} = 2.14\times10^{29}\) J.

(b) \(K = \frac{1}{2} M \left(\frac{2\pi R}{7}\right)^2 = \frac{2\pi^2 (5.97\times10^{34}\) kg/(1.50\times10^{11}\) m\(^2\)}{(3.156\times10^7\) s\(^2\)} = 2.66\times10^{33}\) J.

(c) Since the earth’s moment of inertia is less than that of a uniform sphere, more of the earth’s mass must be concentrated near its center.

EVALUATE: These kinetic energies are very large, because the mass of the earth is very large.

Note: \(\omega\) of Earth on axis is counterclockwise observed looking down on N pole; and \(\omega\) of Earth around Sun is also counterclockwise from same perspective.

Thus Earth must make \(l + \frac{1}{365.25}\) revolutions in a day of 24 hr = 86,400 s.

Thus \(T = \frac{86,400\) sec}{l + \frac{1}{365.25}} = 86,164\) s.
A uniform, solid disk with mass $m$ and radius $R$ is pivoted about a horizontal axis through its center. A small object of the same mass $m$ is glued to the rim of the disk. If the disk is released from rest with the small object at the end of a horizontal radius, find the angular speed when the small object is directly below the axis.

**Identify:** Using energy considerations, the system gains as kinetic energy the lost potential energy, $mgR$.

**Set Up:** The kinetic energy is $K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$, with $I = \frac{1}{2}mR^2$ for the disk. $v = Rw$.

**Execute:** $K = \frac{1}{2}I\omega^2 + \frac{1}{2}m(\omega R)^2 = \frac{1}{2}(I + mR^2)\omega^2$. Using $I = \frac{1}{2}mR^2$ and solving for $\omega$, $\omega^2 = \frac{4g}{3R}$ and $\omega = \sqrt{\frac{4g}{3R}}$.

**Evaluate:** The small object has speed $v = \sqrt{\frac{2gR}{3}}$. If it was not attached to the disk and was dropped from a height $h$, it would attain a speed $\sqrt{2gh}$. Being attached to the disk reduces its final speed by a factor of $\sqrt{\frac{2}{3}}$.

**Initial**

![Initial Diagram]

$m\quad m\quad \omega_0 = 0$

**Final**

![Final Diagram]