Problem 19.53

A monatomic ideal gas expands slowly to twice its original volume, doing 550 J of work in the process.

Part A

Find the heat added to the gas if the process is isothermal. Express your answer with the appropriate units. ANSWER:

\[ Q = \text{______________} \]

Part B

Find the change in internal energy of the gas if the process is isothermal. By above, \( \Delta U = u_2 - u_1 = 0.0 \)

Part C

Find the heat added to the gas if the process is adiabatic. Express your answer with the appropriate units. ANSWER:

\[ Q = \text{______________} \]

Part D

Find the change in internal energy of the gas if the process is adiabatic. By 1st Law \( \Delta U = Q - W \). But \( Q = 0 \). So \( \Delta U = -W = -550 \text{J} \)

Part E

Find the heat added to the gas if the process is isobaric. Express your answer to two significant figures and include the appropriate units.

ANSWER:
Part F

Find the change in internal energy of the gas if the process is isobaric.

Express your answer to two significant figures and include the appropriate units.

ANSWER:

\[ U_2 - U_1 = \]

Part E

Isobaric, ie \( p = \text{constant} \)

For isobaric process

\[ Q = nC_p(T_2 - T_1) \]

By ideal gas \( pV = nRT \) \( \rightarrow T = \frac{pV}{nR} \)

So \( Q = nC_p \left( \frac{p_2V_2}{nR} - \frac{p_1V_1}{nR} \right) \)

But \( p_2 = p_1 = p \)

\[ Q = \frac{C_p}{R} p (V_2 - V_1) \]

For isobaric process \( W = p(V_2 - V_1) \)

Thus \( Q = \frac{C_p}{R} W \)

For monatomic ideal gas \( C_p = \frac{5}{2} R \)

Thus \( Q = \frac{5}{2} W = \frac{5}{2}(550 \text{ J}) = 1375 \text{ J} \)

Part F

1st Law

\[ \Delta U = Q - W \]

\[ \Delta U = 1375 - 550 = 825 \text{ J} \]

Note: In all cases the work is the shaded area, i.e. \( W = \int p \, dV \) (The three p-V diagrams are not to the same scale.)
Problem 19.56

A cylinder with a piston contains 0.146 mol of nitrogen at a pressure of 1.74 \times 10^5 \text{ Pa} and a temperature of 300 \text{ K}. The nitrogen may be treated as an ideal gas. The gas is first compressed isobarically to half its original volume. It then expands adiabatically back to its original volume, and finally it is heated isochorically to its original pressure.

**Part A**

Compute the temperature at the beginning of the adiabatic expansion.

**ANSWER:**

\[ T_1 = \underline{300} \text{ K} \]

\[ T_1 = \frac{300 \text{K} \left( \frac{V_0/2}{V_0} \right)}{\underline{150} \text{K}} = \underline{150} \text{K} \]

**Part B**

Compute the temperature at the end of the adiabatic expansion.

**ANSWER:**

\[ T_2 = \underline{114} \text{ K} \]

\[ 1 \rightarrow 2 \text{ Adiabatic (} Q = 0 \text{)} \]

\[ \frac{T_1}{V_1^{y-1}} = \text{Constant} \]

\[ \frac{T_2}{V_2^{y-1}} = T_1 \frac{V_1^{y-1}}{V_2^{y-1}} \rightarrow T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{y-1} \]

Diatomic Ideal Gas

\[ y = \frac{C_p}{C_v} = \frac{7/2 R}{5/2 R} = \frac{7}{5} = 1.4 \]

\[ V_1 = \frac{V_0}{2}, \quad V_2 = V_0 \]

\[ T_2 = 150 \left( \frac{1}{2} \right)^{1.4-1} \rightarrow 150 \left( \frac{1}{2} \right) = 114 \text{ K} \]

**Part C**

Compute the minimum pressure.

**ANSWER:**

\[ P = \underline{6.59 \times 10^4} \text{ Pa} \]

For 1 \rightarrow 2 \text{ Adiabatic:}

\[ p V^y = \text{Constant} \]

\[ p_2 V_2^y = p_1 V_1^y \]

\[ p_2 = p_1 \left( \frac{V_1}{V_2} \right)^y \]

\[ p_2 = \underline{1.74 \times 10^5} \left( \frac{1}{2} \right)^{1.4} \]

\[ = 6.59 \times 10^4 \text{ Pa} \]

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