More Types of Forces

There are more types of forces than just normal and weight forces. But first, all forces have units of Newton \([N]\). I have chosen to name a normal force as \(n\) in free body diagrams and equations so not to confuse it with the unit \([N]\). However you must remain aware of the distinction between the name of a variable and its units. Many symbols will be introduced in a physics course and there will be numerous cases when symbols will be recycled. So unfortunately, Student Beware!!

Let’s look at the unit \([N]\). Since \(F = ma\), the units must satisfy

\[
[N] = [kg][\frac{m}{s^2}] = [\frac{kg \cdot m}{s^2}]
\]

and

\[
[kg] = [\frac{Ns^2}{m}]
\]

You will often need these two equivalences.

Friction forces - These forces occur when an object can slide across a surface. Unlike normal force, friction acts parallel to the surface. There are two kinds, kinetic and static friction. There is never just one and not the other. If friction is to be ignored, both must be. Kinetic friction always acts opposite to the direction of motion. Its value is

\[
F_k = \mu_k N
\]
where $n$ is the value of the normal force at the interface and $\mu_k$ is the dimensionless coefficient of kinetic friction. Typically $0.5 \leq \mu_k \leq 1.0$. Static friction is more difficult to analyse since it occurs when there is no relative motion. It acts to prevent motion. Its value is limited

$$F_s \leq \mu_s n$$

where $\mu_s$ is the coefficient of static friction and typically has a value $\mu_s > \mu_k$. The actual value and direction of $F_s$ is such as to offset other forces which may cause motion.

If there is motion, the friction is the kinetic friction.

Example: A 10 kg mass is at rest on a horizontal surface. It is pushed with a 50N force.

$\mu_s = 0.60$, $\mu_k = 0.50$. Will the mass move?

\[
\begin{align*}
\text{FBD:} & \quad F \quad F_f \\
\text{m} & \quad \text{F} \quad \text{w = mg} \\
\hline
\text{NL1:} \quad \Sigma F_y = 0 \quad \Rightarrow \quad N = mg = 10\,\text{kg} \times 9.8\,\text{m/s}^2 = 98\,\text{N} \\
\text{Thus} \quad F_k = 0.50 \times 98\,\text{N} = 49\,\text{N} \\
F_s \leq 0.60 \times 98\,\text{N} = 59\,\text{N} \\
\text{The mass is at rest so we give $F_s$ a chance to prevent motion. Apply NL1 in the $x$ direction with $F_f = F_s$.} \\
\Sigma F_x = 0 \Rightarrow F - F_s = 0 \\
F_s = F = 50\,\text{N} \\
\text{This satisfies $F_s \leq 59\,\text{N}$, so the actual $F_s$ will be} \\
\end{align*}
\]
be 50N acting in the negative x direction and there will be no motion.

What if instead F = 60N is the push? Then a value F_s = 60N would be needed. Since this exceeds its limit of 59N, static friction cannot prevent motion and we analyse using kinetic friction F_k = F_k.

Then

$$\Sigma F_x = max$$

$$F - F_k = max$$

$$a_x = \frac{60N - 49N}{10kg} = \frac{11N}{10kg} = 1.1m/s^2$$

Be careful to remember to use the correct value of F_k when you have determined that static friction is not large enough to prevent motion or whenever the problem states that the mass is given to already be moving.

Applied forces - The force F in the above problem is an example of an applied force. It is provided by an external agent.

Tension forces - These forces involve a string, rope, wire or cable. In all cases the tension force acts along the direction of the string and of course can only pull, never push. In this course the string will be massless so the tension is the same anywhere along a single string.
Example: \[ \begin{align*} 
M_2 & \quad M_1 & \quad F = 10 \text{ N} \\
\begin{array}{c}
2.0\text{ kg} \\
3.0\text{ kg}
\end{array} 
\end{align*} \]

Frictionless, \( \mu_s = \mu_k = 0 \)

The connecting string is in tension:

\[ T_2 \quad T_1 \]

Even when accelerating, NL2 gives

\[ T_1 - T_2 = 0 \times a \quad \Rightarrow \quad T_1 = T_2 = T \]

For a massless string, the tension is the same anywhere in the single string. \( T_1 \) and \( T_2 \) are not action/reaction pairs (they act on the same object) but \( T_1 \) has an action/reaction pair acting on the 3.0 kg mass and \( T_2 \) has a pair acting on the 2.0 kg mass.

**FBD for \( m_1 \):**

\[ \begin{array}{c}
y \\
x \\
\uparrow \\
\downarrow \\
T \\
n_1 \\
F \\
w_1
\end{array} \]

**NL2: In x-direction**

\[ F - T = m_1 a_{1x} \]

**FBD for \( m_2 \):**

\[ \begin{array}{c}
y \\
x \\
\uparrow \\
\downarrow \\
T \\
n_2 \\
F \\
w_2
\end{array} \]

**NL2: In x-direction**

\[ T = m_2 a_{2x} \]

So we obtain:

\[ F - T = m_1 a_{1x} \]

\[ T = m_2 a_{2x} \]

Using NL1 in the y-direction only gives the obvious \( n_1 = w_1 \), \( n_2 = w_2 \).
we don't know $T$, $a_1x$ and $a_2x$.
But, the string does not appreciably change its length. Thus $a_1x = a_2x = a_x$. So actually there are two equations and two unknowns. From now on we will be smarter and notice when there is only one value of acceleration.
Often a quick way to solve equations for multi-object problems is to add equations. So adding equations gives:

$$F = (m_1 + m_2)a_x$$

$$a_x = \frac{F}{m_1 + m_2} = \frac{10 \text{N}}{3.0 \text{kg} + 2.0 \text{kg}} = 2.0 \text{m/s}^2$$

Then $T = m_2a_x = 2.0 \text{kg} \times 2.0 \text{m/s}^2 = 4.0 \text{N}$

So of the total applied force of $10 \text{N}$, $6 \text{N}$ is devoted to accelerating the $3 \text{kg}$ mass and the remaining $4 \text{N}$ is transmitted by the string under tension to accelerate the $2 \text{kg}$ mass.

Spring - An ideal spring is massless and changes its length from its natural length $l_0$ whenever a force is applied as shown.

\[ F = k \Delta x \]
F is positive when the spring is stretched \((\Delta x > 0)\) and negative when the spring is compressed \((\Delta x < 0)\). The force produced by the spring is the action/reaction pair to the force \(F\) shown. So we say the force produced by the spring, \(F_{sp}\), is given by:

\[
F_{sp} = -k\Delta x
\]

The \(k\) is called the spring constant and has units \([\frac{N}{m}]\).

Don't get hung up on this negative, a spring opposes being made both longer or shorter than its natural length and does exactly what we have all experienced with springs.

Example: Two masses hang vertically using two identical springs with \(k = 4000 \text{ N/m}\). The springs have an unstretched length of 0.10m. When hanging still, \(k = 4000 \text{ N/m}\) what is the change in length for each spring?

FBD Lower mass \(m_1\): 

\[
F_{sp1} = -k\Delta x
\]

Hanging still \(\Rightarrow v_y = 0\)
NL1: \[ \Sigma F_y = 0 \quad F_{sp1} - W_1 = 0 \]
\[ F_{sp1} = W_1 = m_1 g = 4.0 \text{kg} \times 9.8 \text{m/s}^2 = 39.2 \text{N} \]

It is best to not struggle with the negative in the \( F_{sp} = -k\Delta x \). So use
\[ F_{sp} = k\Delta x \]
\[ \Delta x_1 = \frac{F_{sp1}}{k} = \frac{39.2 \text{N}}{4000 \text{N/m}} = 9.8 \times 10^{-3} \text{m} \]
\[ \Delta x_1 = 0.98 \text{cm} \]

Did the spring get longer or shorter?
Of course longer. So \( \Delta x_1 = +0.98 \text{cm} \)
If the bottom spring pulls up on the 4 kg, it must be pulling down on the 6 kg.
So:

FBD Upper mass \( m_2 \): \[ \begin{array}{c}
- \quad \uparrow \quad F_{sp1} \\
\downarrow \quad F_{sp2} \quad \downarrow W_2 = m_2 g
\end{array} \]

NL1: \[ \Sigma F_y = 0 \]
\[ F_{sp2} - W_2 - F_{sp1} = 0 \]
\[ F_{sp2} = W_2 + F_{sp1} = 6.0 \text{kg} \times 9.8 \text{m/s}^2 + 39.2 \text{N} \]
\[ = 98.0 \text{N} \]
\[ \Delta x_2 = \frac{98.0 \text{N}}{4000 \text{N/m}} = 2.45 \times 10^{-2} \text{m} \]
\[ = 2.45 \text{cm} \]

This also is obviously a positive stretch.
The top spring must stretch enough to support the weight of both masses and the bottom spring only stretches sufficient to support the weight of the bottom mass.
Problems for you

The top mass can not move. The lower mass is initially at rest. You increase $F$, starting from zero, until the 20kg mass just begins to move. What is that value of $F$ and also $T$? You maintain the same $F$ value, what is the acceleration of the 20kg mass and what is the value of $T$?

The mass can only move vertically and without friction. When the mass is held with $x=0$, both springs are at their natural length. When the mass is released and stops moving, what is its location $x$?

$M = 10\text{kg}$, $k_1 = 500 \text{N/m}$, $k_2 = 1000 \text{N/m}$