Uniform Circular Motion

The vector acceleration is the time rate of change of the vector velocity.
\[ \vec{a} = \frac{\text{d}}{\text{d}t} \vec{v} \]

There is acceleration anytime the magnitude of velocity is changing or when the direction of velocity is changing or when both are changing.

Uniform circular motion consists of moving in a circle with constant (uniform) velocity. Thus uniform circular motion has an acceleration due to the velocity changing direction. The acceleration is called centripetal acceleration and points radially towards the center of the circle and has magnitude
\[ a_r = \frac{v^2}{R} \]

where \( R \) = radius of circular path and \( v = |\vec{v}| \).

Example: Riding a Ferris Wheel
Let the above diagram represent a ferris wheel of radius \( R = 5.0 \text{ m} \) oriented vertically.
What would be the velocity \( v \) if you are in free fall when at the top of the wheel? What would your "apparent weight" be then when at the bottom of the wheel?

Remember, in free fall, weight is the only force acting on your body. Thus there is no normal force between you and the seat.

**Free Body Diagram at the top**

\[
\begin{align*}
\text{forces:} & \quad \vec{F} = \vec{mg} \\
\text{acceleration:} & \quad \vec{a} = \frac{\vec{v}^2}{R} \\
\text{equations:} & \quad a_y = -\frac{v^2}{R} \\
& \quad \sum F_y = ma_y \\
& \quad -mg = ma_y = -m \frac{v^2}{R} \\
& \quad v^2 = Rg \\
& \quad v = \sqrt{Rg} = \sqrt{5.0 \text{m} \times 9.8 \text{m/s}^2} = 7.0 \text{m/s} \\
\end{align*}
\]

The time to make one complete revolution is called the period, \( T \). In this case the period is the circumference divided by the magnitude of velocity.

\[
T = \frac{2\pi R}{v} = \frac{2\pi \times 5.0 \text{m}}{7.0 \text{m/s}} = 4.5 \text{s}
\]
Free Body Diagram at the bottom

\[ \begin{align*}
\begin{array}{rll}
\text{Free Body Diagram at the bottom} & \\
\begin{array}{c}
y \uparrow \\
\text{a} \quad \text{a}_x = \frac{v^2}{R} \\
\text{a}_y = a_r = \frac{v^2}{R}
\end{array}
\end{array}
\end{align*} \]

The only difference is that the acceleration \( a_r \) is now up (always towards the center of the circular path). Also a normal force \( N \).

**NL2**: \( \Sigma F_y = m a_y \)

\[ N - mg = m \frac{v^2}{R} \]

\[ n = mg + m \frac{v^2}{R} = m \left( 9.8 \text{m/s}^2 + \frac{(7.0 \text{m/s})^2}{5.0 \text{m}} \right) \]

\[ = m(19.6 \text{ m/s}^2) \]

\[ = 2mg = 2w \]

The seat exerts a force \( 2w \) on you. Your "apparent weight" is twice your normal weight.

Now a problem for you.

You are on a roller coaster ride. While moving at \( 8.0 \text{ m/s} \) you traverse a circular section of track with a local radius such that your body follows a circular path of radius \( 5.0 \text{ m} \). Is your seat belt needed to stay in contact with your seat?