Combining Springs

A single spring

\[ F = k \Delta x \]

F is force applied to spring to cause \( \Delta x \) increase of length

Parallel connection of two springs

\[ F = F_1 + F_2 = k_1 \Delta x + k_2 \Delta x \]

\[ F = (k_1 + k_2) \Delta x \]

same as

\[ k_{eq} = k_1 + k_2 \]

Have Common \( \Delta x \), Forces sum

Series connection of two springs

\[ \Delta x = \frac{F}{k_1} + \frac{F}{k_2} \]

\[ \Delta x = \left( \frac{1}{k_1} + \frac{1}{k_2} \right) F \]

same as

\[ \Delta x = \frac{1}{k_{eq}} F \]

\[ \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} \]

Have Common \( F \), \( \Delta x \)'s sum

\[ k_{eq} = \frac{k_1 k_2}{k_1 + k_2} \]
Examples

Parallel

\[ k_1 = 2 \text{ N/m} \quad k_2 = 3 \text{ N/m} \]

\[ k_{eq} = k_1 + k_2 = 5 \text{ N/m} \]

Same as

Series

\[ k_1 = 2 \text{ N/m} \quad k_2 = 3 \text{ N/m} \]

\[ k_{eq} = \frac{k_1 \cdot k_2}{k_1 + k_2} \]

\[ k_{eq} = \frac{2 \text{ N/m} \cdot 3 \text{ N/m}}{2 \text{ N/m} + 3 \text{ N/m}} \]

\[ k_{eq} = \frac{6}{5} \text{ N/m} \]

Same as

\[ k_{eq} = \frac{6}{5} \text{ N/m} \]