Math 34A Practice Final

1) Use the graph of \( y = 10^x \) to find approximate values:
   a) \( 50^{0.3} \)
   b) \( y'(0.65) \)

2) Integrate:
   a) \( \int \left( 5e^{mx} + \frac{6}{\sqrt{x}} \right) dx \)
   b) \( \int_{a}^{b} (x^3 - 6kx) \, dx \)
   c) \( \int_{\ln(2)}^{\ln(0.5)} \left( e^t - e^{-t} \right) \, dt \)

3) The population of a certain town is increasing at a rate of \( 5 + 2x \) people per month (\( x \) represents the number of months). If the population is currently 23,000 people, by how much will the population increase during the next 8 months?

4) Find values of \( x \) where the function \( f(x) = x^4 - 4x^3 + 4 \)
   a) is increasing
   b) is concave down
   c) has a minimum point

5) A theater currently sells tickets at a price of $8, and sells 1000 tickets. It is estimated that for each 10-cent increase in the price, 20 fewer tickets will be sold. What is the optimal price for the theater to charge (to maximize their income)?

6) Given the two functions \( f(x) = 3x^2 - 8x \) and \( g(x) = -2x^2 + 5 \)
   a) Find the equation of the line tangent to \( f(x) \) at \( x=2 \)
   b) For which value of \( x \) is the slope of \( f(x) \) equal to the slope of \( g(x) \)?

7) A rock is thrown from the top of a 300-meter high cliff. Its velocity (in meters per second) is given by the formula \( v(t) = 20 - 10t \), where \( t \) is the time in seconds after the rock is thrown.
   a) When does the rock hit the ground?
   b) What is the speed of the rock at the time found in part a)?
8) The table gives the position of a car at various times during an afternoon drive.

<table>
<thead>
<tr>
<th>Time</th>
<th>1 pm</th>
<th>1:30 pm</th>
<th>2 pm</th>
<th>3 pm</th>
<th>4 pm</th>
<th>4:30 pm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>0 mi.</td>
<td>20 mi.</td>
<td>45 mi.</td>
<td>70 mi.</td>
<td>105 mi.</td>
<td>120 mi.</td>
</tr>
</tbody>
</table>

a) Find the average speed of the car between 2pm and 4pm.
b) During what time interval was the average speed greatest?

9) A cylindrical container with no top is to be made to hold a volume of $12\pi$ cm$^3$. Material for the side costs 2 cents per cm$^2$ and the material for the bottom costs 3 cents per cm$^2$. What are the dimensions of the least expensive container?

Radius = _____ cm
Height = _____ cm

10) The rate (in kilowatts) at which power is used in a household is shown in the graph below. Find the total energy used between 12pm and 6pm.

Answer: ________ kilowatt-hours

11) Given $m(t) = 3e^{5t} - k^{2t} + t^2$

a) Find $m'(t)$
b) Find $m''(2)$
Math 34A Practice Final – Answers

1a) 3.2
1b) 10.3

2a) \( \frac{5e^{\pi x}}{\pi} + 12\sqrt[3]{x} + C \)
2b) \( \frac{b^4 - a^4}{4} + 3k(a^2 - b^2) \)
2c) 0

3) 104 people

4a) f(x) increasing for x>3
4b) f(x) concave down for 0<x<2
4c) f(x) has a minimum when x=3

5) $6.50

6a) y=4x-12
6b) x=4/5

7a) 10 sec
7b) 80 m/s

8a) 30 mph
8b) 1:30pm to 2pm (avg speed is 50mph)

9) Radius = 2cm
   Height = 3cm

10) 12.5 kilowatt-hours

11a) m’(t) = 15e^{5t} – 2t^3
11b) m''(2) = 75e^{10} + 3/8