Physics 6C

Interference of EM Waves
Constructive Interference:

Waves add - larger amplitude. These waves are “In Phase”

Destructive Interference:

Waves cancel - smaller amplitude. These waves are “Out of Phase” They are out of sync by $\frac{1}{2}\lambda$

Interference in action:
http://phys23p.sl.psu.edu/phys_anim/waves/embeder1.203.html
Here’s a sample problem involving sound waves:

Identical speakers A and B are 4m apart and emit the same constant frequency of 250 Hz from the same source. Find the place(s) in between the speakers that have a loud sound (constructive interference). Also find position(s) where the sounds gets quieter (destructive). Assume the speed of sound in air is 340 m/s.
In Young’s double-slit experiment light comes from the left and passes through the slits, illuminating the screen some distance $R$ away. The light rays from the 2 slits will travel different distances to get to the screen (except in the center). Depending on the path length difference the waves will be in phase or out of phase when they arrive at the screen. If they are in phase, they combine to give constructive interference (a bright region). Out of phase means destructive interference (dark region). Some geometry gives us a formula for this difference in path length: $d \sin(\theta)$. This yields the following formulas:

$$d \cdot \sin(\theta) = m\lambda \leftrightarrow \text{constructive}$$

$$m \text{ can be any integer}$$

$$d \cdot \sin(\theta) = (m + \frac{1}{2})\lambda \leftrightarrow \text{destructive}$$

$$m = 0, 1, \pm 2, \pm 3, \pm 4, \ldots$$

$$\tan(\theta) = \frac{y}{R} \quad y = \text{actual distance on screen (from center)}$$
Here’s a sample problem:

Light with wavelength 546nm passes through two slits and forms an interference pattern on a screen 8.75m away. If the linear distance on the screen from the central fringe to the first bright fringe above it is 5.16cm, what is the separation of the slits?
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This should be a straightforward problem. We are given enough information to just use our formulas. Start with the formula involving the distance on the screen:

\[
\tan(\theta) = \frac{y}{R}
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\tan(\theta) = \frac{y}{R}
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\[
\tan(\theta) = \frac{5.16 \times 10^{-2} \text{m}}{8.75 \text{m}} = 5.897 \times 10^{-3}
\]

\[
\theta = 0.338^\circ
\]
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Use this angle in the formula for bright fringes, with \(m=1\)

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\[ \theta = 0.338^\circ \]

Use this angle in the formula for bright fringes, with m=1

\[ d \cdot \sin(\theta) = m \lambda \]

\[ d \cdot \sin(0.338^\circ) = 546 \cdot 10^{-9} \text{m} \]

\[ d = 9.26 \cdot 10^{-5} \text{m} \]

Since the angle was small we could have used the approximate formula:

\[ y_m = R \frac{m \cdot \lambda}{d} \]
Multiple Slits (diffraction gratings)

These work just like the double slit experiment (same formula), but the bright spots are narrower, and the dark spots are wider. If the grating has more slits the result is a sharper image, with narrower bright fringes.

Two slits produce one minimum between adjacent maxima.

Eight slits produce larger, narrower maxima in the same locations, separated by seven minima.

With sixteen slits, the maxima are still taller and narrower, with more intervening minima.
Single Slit Diffraction

- Similar to the double-slit experiment.
- The formulas are opposite (the geometry just comes out that way).
- Notice that the central maximum is double-width compared to the others.
- This is how you can tell a single-slit pattern from a multiple-slit pattern.

**Formulas for Destructive Interference (dark fringes)**

\[ a \cdot \sin(\theta) = (m) \cdot \lambda \]

\[ y_m = R \frac{(m) \cdot \lambda}{a} \]

These approximate formulas work when the angle is small.

**Formulas for Constructive Interference (bright fringes)**

\[ a \cdot \sin(\theta) = (m + \frac{1}{2}) \cdot \lambda \]

\[ y_m = R \frac{(m + \frac{1}{2}) \cdot \lambda}{a} \]
Here’s a sample problem:

How many dark fringes will be produced on either side of the central maximum if green light ($\lambda=553\text{nm}$) is incident on a slit that is $2\mu\text{m}$ wide?
Here’s a sample problem:

How many dark fringes will be produced on either side of the central maximum if green light ($\lambda=553\text{nm}$) is incident on a slit that is 2µm wide?

This is a single-slit problem, so the formula for the dark fringes is:  

$$a \cdot \sin(\theta) = m\lambda$$
Here’s a sample problem:

How many dark fringes will be produced on either side of the central maximum if green light \((\lambda=553\text{nm})\) is incident on a slit that is 2µm wide?

This is a single-slit problem, so the formula for the dark fringes is: \(a \cdot \sin(\theta) = m\lambda\)

Let’s find the angles to the first few dark fringes. We get a new angle for each value of \(m\).

<table>
<thead>
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When we try to calculate with \(m=4\) we get a calculator error. Why doesn’t it work?
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When we try to calculate with $m=4$ we get a calculator error. Why doesn’t it work?

Recall the single-slit diffraction diagram.

For the fringes to show up on the screen, the angle must be less than 90°.

Of course the pattern gets very dim near the edges, but mathematically the formula will break down when $\sin(\theta)>1$. 

Prepared by Vince Zaccone
For Campus Learning Assistance Services at UCSB
Here’s a sample problem:

How many dark fringes will be produced on either side of the central maximum if green light (\( \lambda = 553 \text{nm} \)) is incident on a slit that is 2\( \mu \text{m} \) wide?

This is a single-slit problem, so the formula for the dark fringes is:  
\[
\theta = m \frac{\lambda}{a} 
\]

Let’s find the angles to the first few dark fringes. We get a new angle for each value of \( m \).

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When we try to calculate with \( m = 4 \) we get a calculator error. Why doesn’t it work?

Recall the single-slit diffraction diagram.

For the fringes to show up on the screen, the angle must be less than 90°.

Of course the pattern gets very dim near the edges, but mathematically the formula will break down when \( \sin(\theta) > 1 \).

So it looks like we will get 3 dark fringes.
Thin Film Interference

Light reflected from the upper and lower surfaces of the film comes together in the eye at P and undergoes interference.

Some colors interfere constructively and others destructively, creating the iridescent color bands we see.

Basic idea is that we will compare the two reflections.
If they are in phase we have constructive interference (bright).
If they are out of phase we have destructive interference (dark).

Important details:
1) When light reflects from a higher-index medium it is phase-shifted by ½ of a wavelength. If both reflected rays have this shift we can ignore it, but if only one of them is shifted, we have to switch the formulas for constructive/destructive interference.
2) The wavelength in the formulas is the wavelength in the film, so we have to divide the vacuum wavelength by the index of the film.

Thin film demo

FORMULAS – no relative shift
2t = mλ ↔ constructive
2t = (m + ½)λ ↔ destructive
Here's a sample problem:

a) What is the minimum soap-film thickness (n=1.33) in air that will produce constructive interference in reflection for red (λ=652nm) light? (assume normal incidence)

b) Which visible wavelength(s) will destructively interfere when reflected from a soap film of thickness 613nm? Assume a range of 350nm to 750nm for visible light.
Here’s a sample problem:

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a) What is the minimum soap-film thickness \( (n=1.33) \) in air that will produce constructive interference in reflection for red \( (\lambda=652\text{nm}) \) light? (assume normal incidence)

b) Which visible wavelength(s) will destructively interfere when reflected from a soap film of thickness 613nm? Assume a range of 350nm to 750nm for visible light.

The outgoing rays will interfere, and there is a relative phase shift, since ray 1 reflects from a higher index, but ray 2 does not.

This yields the following formulas:

\[
m \cdot \frac{\lambda_0}{n} = 2 \cdot t \quad \text{destructive}
\]

\[
(m + \frac{1}{2}) \cdot \frac{\lambda_0}{n} = 2 \cdot t \quad \text{constructive}
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For part a) we use the constructive formula, with m=0 (we want the thinnest film possible)

\[ \frac{1}{2} \cdot \left( \frac{652\text{nm}}{1.33} \right) = 2 \cdot t \Rightarrow t_{\text{min}} = 123\text{nm} \]

The outgoing rays will interfere, and there is a relative phase shift, since ray 1 reflects from a higher index, but ray 2 does not.

This yields the following formulas:

\[
\begin{align*}
\frac{m}{n} \cdot \frac{\lambda_0}{n} &= 2 \cdot t \leftarrow \text{destructive} \\
(m + \frac{1}{2}) \cdot \frac{\lambda_0}{n} &= 2 \cdot t \leftarrow \text{constructive}
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For part b) we use the destructive formula, with different values of \(m\) (we want visible wavelengths)
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\text{For part b)} & \quad \text{we use the destructive formula, with different values of } m \quad (\text{we want visible wavelengths}) \\
m \cdot \frac{\lambda_0}{n} = 2 \cdot t \quad \Rightarrow \quad \lambda_0 = \frac{2 \cdot t \cdot n}{m}
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For part b) we use the destructive formula, with different values of \(m\) (we want visible wavelengths)

\[
m \cdot \frac{\lambda_0}{n} = 2 \cdot t \Rightarrow \lambda_0 = \frac{2 \cdot t \cdot n}{m}
\]

\[
m = 3 \Rightarrow \lambda_0 = 544\text{nm}
\]

\[
m = 4 \Rightarrow \lambda_0 = 407\text{nm}
\]

Other values of \(m\) give wavelengths that fall outside of the visible range.
Here’s a sample problem:

A thin layer of magnesium fluoride (n=1.38) is used to coat a flint-glass lens (n=1.61).

What thickness should the MgF₂ coating be to suppress the reflection of 595nm light?
Here’s a sample problem:

A thin layer of magnesium fluoride \((n=1.38)\) is used to coat a flint-glass lens \((n=1.61)\).

What thickness should the \(\text{MgF}_2\) coating be to suppress the reflection of 595nm light?
Here’s a sample problem:

A thin layer of magnesium fluoride (n=1.38) is used to coat a flint-glass lens (n=1.61).

What thickness should the MgF$_2$ coating be to suppress the reflection of 595nm light?

We need destructive interference (no reflection). In this case both outgoing rays reflect from a higher index, so there is no relative phase shift.

Our formulas are:

\[
\frac{m \cdot \lambda_0}{n} = 2 \cdot t \quad \text{constructive}
\]

\[
(m + \frac{1}{2}) \cdot \frac{\lambda_0}{n} = 2 \cdot t \quad \text{destructive}
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(m + \frac{1}{2}) \cdot \frac{\lambda_0}{n} = 2 \cdot t \quad \text{destructive}
\]

We can use any integer $m > 0$, so start with $m=0$ and solve for $t$.

This will give the minimum thickness.

\[
(0 + \frac{1}{2}) \cdot \frac{595\text{nm}}{1.38} = 2 \cdot t \quad \Rightarrow t_{\text{min}} = 108\text{nm}
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\]

To get other possible thicknesses that will work, just use larger values for \(m\):

- \(m = 1 \Rightarrow t = 323\text{nm}\)
- \(m = 2 \Rightarrow t = 539\text{nm}\)
- etc..
Here’s a sample problem:

A thin film of oil (n=1.40) floats on water (n=1.33). When sunlight is incident vertically, the only colors that are absent from the reflected light are blue (458nm) and red (687nm).

Estimate the thickness of the film.
Here’s a sample problem:

A thin film of oil (n=1.40) floats on water (n=1.33). When sunlight is incident vertically, the only colors that are absent from the reflected light are blue (458nm) and red (687nm).

Estimate the thickness of the film.
Here's a sample problem:

A thin film of oil (n=1.40) floats on water (n=1.33). When sunlight is incident vertically, the only colors that are absent from the reflected light are blue (458nm) and red (687nm).

Estimate the thickness of the film.

In this case ray 1 reflects from a higher index, but ray 1 reflects from a lower index, so there is a relative phase shift.

Our formulas are:

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m \cdot \frac{\lambda_0}{n} = 2 \cdot t \quad \text{destructive}
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![Diagram of light rays in oil and water layers]

In this case ray 1 reflects from a higher index, but ray 1 reflects from a lower index, so there is a relative phase shift.

Our formulas are:

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\text{destructive:} & \quad m \cdot \frac{\lambda_0}{n} = 2 \cdot t \\
\text{constructive:} & \quad (m + \frac{1}{2}) \cdot \frac{\lambda_0}{n} = 2 \cdot t
\end{align*}
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We will use the destructive interference formula for each given wavelength. Since they are the only visible wavelengths that are absent, we can deduce that they correspond to consecutive values for m in the formula.
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Estimate the thickness of the film.

![Diagram of light rays reflecting from air, oil, and water layers]

In this case ray 1 reflects from a higher index, but ray 1 reflects from a lower index, so there is a relative phase shift.

Our formulas are:

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m \cdot \frac{\lambda_0}{n} = 2 \cdot t \quad \text{destructive}
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(m + \frac{1}{2}) \cdot \frac{\lambda_0}{n} = 2 \cdot t \quad \text{constructive}
\]

We will use the destructive interference formula for each given wavelength. Since they are the only visible wavelengths that are absent, we can deduce that they correspond to consecutive values for m in the formula.

\[
m \cdot \frac{687\text{nm}}{1.40} = 2 \cdot t
\]

\[
(m + 1) \cdot \frac{458\text{nm}}{1.40} = 2 \cdot t
\]
Here’s a sample problem:

A thin film of oil (n=1.40) floats on water (n=1.33). When sunlight is incident vertically, the only colors that are absent from the reflected light are blue (458nm) and red (687nm).

Estimate the thickness of the film.

In this case ray 1 reflects from a higher index, but ray 2 reflects from a lower index, so there is a relative phase shift.

Our formulas are:

\[ m \cdot \frac{\lambda_0}{n} = 2 \cdot t \quad \text{← destructive} \]

\[ (m + \frac{1}{2}) \cdot \frac{\lambda_0}{n} = 2 \cdot t \quad \text{← constructive} \]

We will use the destructive interference formula for each given wavelength. Since they are the only visible wavelengths that are absent, we can deduce that they correspond to consecutive values for \( m \) in the formula.

\[ m \cdot \frac{687\text{nm}}{1.40} = 2 \cdot t \]

\[ (m + 1) \cdot \frac{458\text{nm}}{1.40} = 2 \cdot t \]

At this point we have some algebra to do. My preference is to find the integer value of \( m \) that fits the formulas, then plug that in to find \( t \).
Here’s a sample problem:

A thin film of oil \((n=1.40)\) floats on water \((n=1.33)\). When sunlight is incident vertically, the only colors that are absent from the reflected light are blue \((458\text{nm})\) and red \((687\text{nm})\).

Estimate the thickness of the film.

![Image of light rays](image)

In this case ray 1 reflects from a higher index, but ray 1 reflects from a lower index, so there is a relative phase shift.

Our formulas are:

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\begin{align*}
    m \cdot \frac{\lambda_0}{n} &= 2 \cdot t \quad \text{destructive} \\
    (m + \frac{1}{2}) \cdot \frac{\lambda_0}{n} &= 2 \cdot t \quad \text{constructive}
\end{align*}
\]

We will use the destructive interference formula for each given wavelength. Since they are the only visible wavelengths that are absent, we can deduce that they correspond to consecutive values for \(m\) in the formula.

\[
\begin{align*}
    m \cdot \frac{687\text{nm}}{1.40} &= 2 \cdot t \\
    (m + 1) \cdot \frac{458\text{nm}}{1.40} &= 2 \cdot t
\end{align*}
\]

At this point we have some algebra to do. My preference is to find the integer value of \(m\) that fits the formulas, then plug that in to find \(t\).

\[
\begin{align*}
    m \cdot \frac{687\text{nm}}{1.40} &= (m + 1) \cdot \frac{458\text{nm}}{1.40} \\
    (m)(687) &= (m + 1)(458) \\
    (m)(687) &= (m)(458) + (1)(458) \\
    229m &= 458 \Rightarrow m = 2
\end{align*}
\]
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A thin film of oil (n=1.40) floats on water (n=1.33). When sunlight is incident vertically, the only colors that are absent from the reflected light are blue (458nm) and red (687nm).

Estimate the thickness of the film.

![Diagram](image)

In this case ray 1 reflects from a higher index, but ray 2 reflects from a lower index, so there is a relative phase shift.

Our formulas are:

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\frac{m \cdot \lambda_0}{n} & = 2 \cdot t \quad \text{destructive} \\
(m + \frac{1}{2}) \cdot \frac{\lambda_0}{n} & = 2 \cdot t \quad \text{constructive}
\end{align*}
\]

We will use the destructive interference formula for each given wavelength. Since they are the only visible wavelengths that are absent, we can deduce that they correspond to consecutive values for m in the formula.

\[
m \cdot \frac{687\text{nm}}{1.40} = 2 \cdot t
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\[
(m + 1) \cdot \frac{458\text{nm}}{1.40} = 2 \cdot t
\]

At this point we have some algebra to do. My preference is to find the integer value of m that fits the formulas, then plug that in to find t.

\[
m \cdot \frac{687\text{nm}}{1.40} = (m + 1) \cdot \frac{458\text{nm}}{1.40}
\]

\[
(m) (687) = (m + 1) (458)
\]

\[
(m) (687) = (m) (458) + (1) (458)
\]

\[
229m = 458 \Rightarrow m = 2
\]

\[
2 \cdot \frac{687\text{nm}}{1.40} = 2 \cdot t \Rightarrow t = 491\text{nm}
\]
Link to a cool butterfly wing animation