Physics 6A

Angular Momentum
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If we simply translate the variables, we can get one possible equation:

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If we simply translate the variables, we can get one possible equation:

\[ \vec{p} = m \cdot \vec{v} \]  
This is the formula for linear momentum

\[ \vec{L} = I \cdot \vec{\omega} \]  
Using \( I \) instead of \( m \) and \( \vec{\omega} \) instead of \( \vec{v} \), we get this formula for angular momentum

Note that momentum is a vector quantity. We probably will not need to consider that aspect of it in this class, however. If you are curious about what you are missing, check out the section in your book about gyroscopes. Good stuff.
Angular Momentum

Consider the simple case of a small mass (m) tied to a string with radius r. If the mass is swung around in a circle it will have some angular velocity $\omega$.

Notice that it will also have linear velocity (tangential to the circle). The relationship we know for these is $v = r\omega$.

We can use this idea to find a useful alternate formula for angular momentum.

The point mass will have a moment of inertia:

$$I = m \cdot r^2$$

Substituting into the standard formula:

$$L = I \cdot \omega$$

$$L = (m \cdot r^2) \cdot \left(\frac{v}{r}\right)$$

$$L_{\text{point mass}} = m \cdot v \cdot r$$

We can use this formula when we have a point mass with a given linear velocity at some distance from a pivot point or axis of rotation.
Angular Momentum

Like linear momentum, angular momentum is **conserved**. We will use this concept in several types of problems.

$$I_f \cdot \omega_f = I_i \cdot \omega_i$$

This is a formula for conservation of angular momentum.
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This is a formula for conservation of angular momentum.

Also, we should have some formula relating angular momentum to torque (just like we have a formula relating linear momentum to force):

\[ \Delta \vec{L} = \vec{\tau} \cdot \Delta t \]

This says that any time a torque is applied, there will be a corresponding change in angular momentum.

Again, we probably won’t need the vector aspect here, but we might need +/- signs to keep track of it.
Example: A 28 kg child is sitting on the edge of a 145 kg merry-go-round of radius 2.5 m while it is spinning at a rate of 3.7 rpm. If the child moves to the center, how fast will it be spinning? Assume the merry-go-round is a uniform cylinder.
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We can use conservation of momentum for this one. Initially both the merry-go-round and the child contribute to the total angular momentum. However, once the child is at the center, she no longer has angular momentum ($r=0$).
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\[ L_f = L_i \]

We need the total angular momentum of the system, including both the child and the disk.

\[ I_{\text{disk},f} \cdot \omega_{\text{disk},f} = I_{\text{disk},i} \cdot \omega_{\text{disk},i} + I_{\text{girl},i} \cdot \omega_{\text{girl},i} \]
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The initial angular velocity is given in rpm, and we can leave it in those units for this type of problem.
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\[
\left( \frac{1}{2} m_{\text{disk}} r^2 \right) \cdot \omega_{\text{disk}} f = \left( \frac{1}{2} m_{\text{disk}} r^2 \right) \cdot (\omega_i) + \left( m_{\text{girl}} r^2 \right) \cdot (\omega_i)
\]

\[
\left( \frac{1}{2} \cdot 145 \text{ kg} \cdot (2.5 \text{ m})^2 \right) \cdot \omega_{\text{disk}} f = \left( \frac{1}{2} \cdot 145 \text{ kg} \cdot (2.5 \text{ m})^2 \right) \cdot (3.7 \text{ rpm}) + (28 \text{ kg} \cdot (2.5 \text{ m})^2) \cdot (3.7 \text{ rpm})
\]

\[
\omega_{\text{disk}} f = 5.1 \text{ rpm}
\]
Example: A helicopter rotor blade can be considered a long thin rod, as shown in figure below. Each of the three rotor helicopter blades is $L = 3.6$ m long and has a mass of $m = 150$ kg. While starting up, the motor provides a constant torque for 2 minutes, and the blades speed up from rest to 85 rad/s.

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We will use angular momentum for this one. Since we have a constant torque, we simply need to find the change in angular momentum and divide by time.

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We need to find the moment of inertia for the blades. There are 3 of them, and each one is a long thin rod.

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To find the speed of the blade tips, simply use \( v = r\omega \):

\[
v = (3.6 \text{ m}) \cdot (85 \text{ rad/s}) = 306 \text{ m/s}
\]