1. a) Find the moment of inertia of a rod of length $l$, mass $m$ rotating about an axis a distance $x$ from its midpoint as in the figure:

Hint: the moment of inertia of a rod rotating about its center is $\frac{1}{12}ml^2$

b) Using your result from part a, find the moment of inertia of a square plate of mass $m$, side length $l$ rotating about its center.

2. Forces are applied to opposite corners of the square plate in problem 1, as in the figure. The plate itself rests on a frictionless surface, with a vertical peg through the center to hold it still.

If the plate is initially at rest, calculate:

a) The magnitude and direction of the force exerted by the peg in the center.

b) The net torque about the center of mass of the plate.

c) The angular acceleration about the center of mass.

d) Assuming the angular acceleration remains constant, the time it takes for the plate to complete one revolution about the center of mass.
3. The square plate in the first two problems is
now hung from one of its corners, as in the
figure. It swings back and forth about
the pivot for small angles.

a) calculate the angular frequency,
cyclic frequency, and period
for this physical pendulum.

b) How long should a simple pendulum
be made to achieve the same period?

4. A thin wire of length L, cross-sectional area A,
and Young's modulus y has a large mass M
hung from it.

Assuming the wire
is perfectly elastic, the mass is much
heavier than the wire, and the wire
is unstretched at the
moment the mass is
attached, calculate:

a) The frequency and period
of oscillation of the mass-wire
system.

b) The amplitude of oscillation of the
mass-wire system.

c) The equation of motion of the mass-wire
system, taking \( x = 0 \) to be the location
of the mass when the wire is unstretched,
the downward direction positive.

d) The maximum distance the wire is stretched,
e) The maximum speed of the mass.
f) The maximum acceleration of the mass.
Consider the two arrangements of mass-spring systems depicted below:

I.

\[
\begin{array}{c}
    k_1 \quad M \quad k_2 \quad M
\end{array}
\]

II.

\[
\begin{array}{c}
    k_1 \quad M \quad k_2
\end{array}
\]

In each picture the springs are at their natural lengths. If the mass is pulled a distance \( x \) to the right in each picture calculate:

a) The force exerted on the mass by the springs.

b) The effective spring constant of each combination of springs, if the system is to be modeled as a mass and a single spring.

c) Now repeat parts a and b for the following mass-spring system:

\[
\begin{array}{c}
    k_1 \quad M \quad k_2
\end{array}
\]

6. A damped mass spring system is driven by a sinusoidal force of \( f(t) = 3 \sin(\omega t) \) N, where the frequency of the sinusoidal force can be varied.

The mass is 1 kg, the force constant of the spring is 10 N/m, and the damping coefficient of the spring is 2 Ns/m.

a) Calculate the natural frequency of this system.

b) Calculate the time required for the transient response to die down to 10% of its original amplitude.

c) Calculate the steady state amplitude for angular frequencies of \( \omega = 1, 2, 3, 4, 5 \) radians per second.

d) Calculate the resonant frequency of this system, in other words what should the frequency of \( f(t) \) be to give the largest steady state amplitude?
7. A 70 kg man is walking up a 10 meter ladder of mass 10 kg. The ladder is resting on a wall with no friction, and the base of the ladder is 5 m from the wall. If the coefficient of static friction between the ladder and the floor is $\mu_s = 0.4$, how high can the man walk up the ladder before it begins to slip?

![Diagram of a ladder leaning against a wall with a man walking up it.]

8. The Starship Enterprise is to park in a docking orbit around a planet of mass $M$, a distance $R$ away from the center of mass of the planet.

a) What speed should the Enterprise have in order to attain a circular orbit?

b) If the Enterprise would like to break orbit in order to take leave on the planet Rysea, by what factor should it increase its orbital speed?

c) If the Enterprise is travelling 25% too fast for a circular orbit, it will have an elliptical orbit. What maximum distance will the Enterprise be from the planet if this is the case?

d) What will its minimum speed be?

![Diagram of a planet with an orbit and a spaceship in it.]](image)
A glass with cross-sectional area $A$ is filled to a height of $h$ with water purified by reverse osmosis. An ice cube of side length $l$ is then put into the glass. The density of ice is about 90% of the density of water.

\[ h \quad \rightarrow \quad h' \]

a) How much does the water level rise when the ice cube is put in the glass?

b) When the ice melts, what is the water level?

A cubical block of wood floats at the interface between oil and water with its lower surface 1.5 cm below the interface. The length of each edge of the block is 10 cm. The density of the oil is $900 \, \text{kg/m}^3$.

a) What is the pressure difference between the top of the wood block and the bottom of the wood block?

b) What are the mass and density of the block?
A medical technician is trying to determine what percentage of a patient's artery is blocked by plaque. To do this, she measures the blood pressure just before the region of blockage and finds that it is $1.20 \times 10^4$ Pa, while in the region of blockage it is $1.15 \times 10^4$ Pa. Furthermore, she knows that blood flowing through the normal artery just before the point of blockage is travelling at 30 cm/s, and the specific gravity of the patient's blood is 1.06.

What percentage of the patient's artery is blocked by plaque?