1) Astronaut Benny travels to Vega, the fifth brightest star in the night sky, leaving his 35-year-old twin sister Jenny behind on Earth. Benny travels with a speed of 0.99c, and Vega is 25.3 light-years from Earth.
   a) How long does the trip take from the point of view of Jenny?
   b) How much has Benny aged when he arrives at Vega?

   part a): \[ \Delta t = \frac{d}{v} = \frac{25.3 \text{ light-years}}{0.99c} = 25.6 \text{ yrs} \]

   part b): \[ \Delta t = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} \Rightarrow \Delta t = \sqrt{\frac{1}{1 - \left(0.99^2\right)}} \approx 3.6 \text{ yrs} \]

2) An astronaut moving with a speed of 0.65c relative to Earth measures her heart rate to be 72 beats per minute.
   a) When an Earth-based observer measures the astronaut’s heart rate is the result greater than, less than, or equal to 72 bpm?
   b) Calculate the astronaut’s heart rate as measured on Earth.

   part a): Less than 72bpm – the moving object undergoes time dilation.

   part b): each heartbeat takes 1/72 minutes in the astronaut frame. This is \( \Delta t_0 \).

   \[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \sqrt{\frac{1}{1 - \left(0.65^2\right)}} \approx 0.0183 \text{ min} \Rightarrow 55 \text{ beats per min} \]

3) The Linac portion of the Fermilab Tevatron contains a high-vacuum tube that is 64m long, through which protons travel at an average speed of 0.65c. How long is the Linac tube, as measured in the proton’s reference frame?

   \[ L_0 = 64 \text{ m} \]

   \[ L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = (64 \text{ m}) \sqrt{1 - \left(0.65^2\right)} = 48.6 \text{ m} \]

4) You and a friend travel through space in identical spaceships. Your friend informs you that he has made some measurements and that his ship is 150m long, but yours is only 120m long. From your point of view,
   a) how long is your friend’s ship?
   b) how long is your ship?
   c) what is the speed of your friend’s ship relative to yours?

   part a): your ships are identical, so they are both 150m long (when at rest). When you are moving relative to your friend, he measures your ship to be 120m, so when you measure his ship, you must also get 120m (symmetry).

   part b): your ships are identical, so they are both 150m long (when at rest).

   part c): \[ L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = 120 \text{ m} = (150 \text{ m}) \sqrt{1 - \left(\frac{v}{c}\right)^2} \Rightarrow v = 0.6c \]
5) Two asteroids head straight for the Earth from the same direction. Their speeds relative to the Earth are 0.8c for #1 and 0.6c for #2. Find the speed of asteroid 1 relative to asteroid 2. We can use the formula from the text directly:
\[
v' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{0.8c - 0.6c}{1 - (0.8)(0.6)} = 0.38c
\]

6) Two rocket ships approach Earth from opposite directions, each with a speed of 0.6c relative to Earth. What is the speed of one ship relative to the other?
This one will be trickier to set up. We have 3 objects (the Earth and 2 ships). Defining the Earth’s frame as the "primed" frame, we have \( v' = 0.6c \) for ship #2. Ship #1’s frame is the "non-primed" frame, and there Earth has velocity \( u = 0.6c \). We are looking for \( v \) (the speed of ship #2 in ship #1’s frame). Use the rearranged version of the formula in the text.
\[
v = \frac{v' + u}{1 + \frac{uv}{c^2}} = \frac{0.6c + 0.6c}{1 + (0.8)(0.6)} = 0.88c
\]

7) An object has a total energy that is 4.5 times its rest energy. What is its speed? Rest energy is given by \( E_{\text{rest}} = mc^2 \).
Total energy is \( E_{\text{total}} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \).
\[
E_{\text{total}} = 4.5 \cdot E_{\text{rest}} \Rightarrow \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = 4.5 \cdot mc^2 \Rightarrow v = 0.975c
\]

8) Electrons are accelerated from rest through a potential difference of 276,000 Volts. Assume the rest mass of an electron is 9.11x10^{-31} kg.
a) What is the final speed calculated classically?
b) What is the final speed calculated relativistically?
part a): \( K = \frac{1}{2} mv^2 = 276000eV = 4.4 \cdot 10^{-14} J \Rightarrow v = 3.1 \cdot 10^8 \text{ m/s} \) (Why is this too fast?)
part b): \( E_{\text{total}} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = K + mc^2 = 4.4 \cdot 10^{-14} j + 8.2 \cdot 10^{-14} J \Rightarrow v = 0.76c = 2.28 \cdot 10^8 \text{ m/s} \)

clas.sa.ucsb.edu/vincezaccone