Physics 6B

Practice Midterm Solutions
1. A block of plastic with a density of 920 kg/m$^3$ floats at the interface between oil of density 850 kg/m$^3$ and water of density 1000 kg/m$^3$, as shown. Calculate the percentage of the plastic which is submerged in the water.

The block is floating motionless, so the net force is 0. The total buoyant force includes contributions from the oil and the water, as shown in the free-body diagram.

\[ F_{b, \text{water}} + F_{b, \text{oil}} - \text{weight} = 0 \]

The key to this problem is to write these forces in terms of densities:

- weight = \( mg = \rho_{\text{plastic}} \cdot V \cdot g \)
- \( F_{b, \text{water}} = \rho_{\text{water}} \cdot V_{\text{water}} \cdot g \)
- \( F_{b, \text{oil}} = \rho_{\text{oil}} \cdot V_{\text{oil}} \cdot g = \rho_{\text{oil}} \cdot (V - V_{\text{water}}) \cdot g \)

Here \( V \) is the total volume of the plastic block.

Putting these into our formula, and solving for \( V_{\text{water}} \):

\[
(\rho_{\text{water}} - \rho_{\text{oil}})V_{\text{water}} = (\rho_{\text{plastic}} - \rho_{\text{oil}})V
\]

\[
V_{\text{water}} = \frac{\rho_{\text{plastic}} - \rho_{\text{oil}}}{\rho_{\text{water}} - \rho_{\text{oil}}} \cdot V
\]

\[
V_{\text{water}} = \frac{920 - 850}{1000 - 850} \cdot V = \frac{70}{150} \cdot V
\]

\[ V_{\text{water}} \approx 47\% \cdot V \]
2. A U-shaped tube contains 10 cm of alcohol \( (\rho_a=800 \text{ kg/m}^3) \), 12 cm of oil \( (\rho_o=850 \text{ kg/m}^3) \), and water \( (\rho_w=1000 \text{ kg/m}^3) \) as shown. Find the distance \( h \) between the surface of the alcohol and the surface of the water.

The gauge pressure on either side must balance at the oil/water interface. On the left, we can add the partial pressures from the alcohol and the oil.

\[
\rho_{\text{alc}} \cdot g \cdot (10\text{cm}) + \rho_{\text{oil}} \cdot g \cdot (12\text{cm}) = \rho_{\text{water}} \cdot g \cdot (22\text{cm} - h) \cdot g \\
\rho_{\text{alc}} \cdot (10\text{cm}) + \rho_{\text{oil}} \cdot (12\text{cm}) = \rho_{\text{water}} \cdot (22\text{cm} - h)
\]

...algebra...

\[
h = \frac{\rho_{\text{water}} \cdot (22\text{cm}) - \rho_{\text{alc}} \cdot (10\text{cm}) - \rho_{\text{oil}} \cdot (12\text{cm})}{\rho_{\text{water}}} \\
h = \frac{(1000)(22) - (800)(10) - (850)(12)}{1000} = 3.8\text{cm}
\]
3. An incompressible fluid is flowing in a horizontal pipe. The radius of the pipe gradually decreases from 10 cm to 5 cm. The velocity of the fluid in the narrow portion will be:
   a) twice as large as in the wide section
   b) half as large as in the wide section
   c) the same as in the wide section
   d) four times as large as in the wide section

Continuity dictates that the flow rate is the same throughout the pipe.

We can use the formula \( A_1 \cdot v_1 = A_2 \cdot v_2 \)

Since the cross-section of the pipe is circular,
\[
\pi r_1^2 \cdot v_1 = \pi r_2^2 \cdot v_2
\]

\[
v_2 = \left(\frac{r_1}{r_2}\right)^2 \cdot v_1
\]

\[
v_2 = \left(\frac{10\text{ cm}}{5\text{ cm}}\right)^2 \cdot v_1 = (2)^2 \cdot v_1
\]

\[
v_2 = 4v_1
\]

Answer d)

This is a problem that you can do quickly if you recognize that the velocity is related to the square of the radius. When the radius decreases by a factor of 2, the velocity increases by a factor of 4.
4. A 750g mass attached to a spring of spring constant $k=124 \text{ N/m}$ is at rest on a horizontal frictionless surface. It is struck by a hammer which gives it a speed of $2.76 \text{ m/s}$ directly toward the spring. It does not move any significant distance while being struck. Which of these statements is true?

a) It will oscillate with a frequency of $12.86 \text{ Hz}$ and an amplitude of $0.215 \text{ m}$.

b) It will oscillate with a period of $0.489 \text{ seconds}$ and an amplitude of $0.304 \text{ m}$.

c) It will move a maximum distance of $21.5 \text{ cm}$ from its equilibrium position, and take $0.244 \text{ seconds}$ to first return to the equilibrium position.

d) It will move a total distance of $86 \text{ cm}$ before it first returns to the equilibrium position, and this will take $0.489 \text{ seconds}$.

We can calculate the period, frequency and amplitude from the given initial values:

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{0.75 \text{ kg}}{124 \text{ N/m}}} \approx 0.4887 \text{ sec}$$

$$f = \frac{1}{T} \approx 2.046 \text{ Hz}$$

$$\omega = 2\pi f \approx 12.86 \text{ rad/ sec}$$

$$V_{\text{max}} = A\omega \Rightarrow 2.76 \frac{\text{ m}}{\text{ s}} = A \left(12.86 \frac{\text{ rad}}{\text{ s}}\right) \Rightarrow A \approx 0.215 \text{ m} = 21.5 \text{ cm}$$

The mass will return to the equilibrium position after half of a period, so $0.244 \text{ sec}$.

The amplitude is max distance from equilibrium, so that is $21.5 \text{ cm}$.

Answer c) is correct

Notice that answer a) is not correct because it specifies frequency ($f$), rather than angular frequency ($\omega$).
5. What happens to a simple pendulum’s frequency if both the mass and length are increased?
   a) The frequency increases.
   b) The frequency decreases.
   c) The frequency does not change.
   d) The frequency may increase or decrease; it depends on the length to mass ratio.

The formula for frequency of a simple pendulum is

\[ f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \]

Notice that there is no m in the equation. This means the frequency is not affected by the mass. The length does matter, however. L is in the denominator, so increasing length should decrease frequency.

Answer b)
6. After plunging from a bridge, a 60kg bungee-jumper oscillates up and down, completing one cycle every 2 seconds. If the bungee cord is assumed to be massless and has an unstretched length of 10m, find the spring constant of the bungee cord.

This is just like a typical mass-on-a-spring problem.

The formula for the period of the motion is

\[ T = 2\pi \sqrt{\frac{m}{k}} \]

We can solve this for \( k \), then plug in the given values.

\[ k = m \left( \frac{2\pi}{T} \right)^2 \]

\[ k = (60 \text{kg}) \left( \frac{2\pi}{2 \text{ s}} \right)^2 = 592 \frac{\text{N}}{\text{m}} \]

Answer c)
A pair of speakers is hooked up to a stereo and placed 4m apart. The same sound is emitted by both speakers and it has a frequency of 68Hz. If you stand between the speakers, at what distance from the left speaker would the sound get quiet (destructive interference)? Use 340 m/s for the speed of sound.

Destructive interference will occur when the distance to one speaker is \( \frac{1}{2} \) wavelength more than the distance to the other one. We can write a formula for this condition, then rearrange to find the distance:

\[
x_1 - x_2 = \frac{1}{2}\lambda
\]

\[
x_1 - (4m - x_1) = \frac{1}{2}\lambda
\]

\[
2x_1 = 4m + \frac{1}{2}\lambda
\]

\[
x_1 = 2m + \frac{1}{4}\lambda
\]

Now we need to find the wavelength:

\[
v_{\text{sound}} = \lambda \cdot f
\]

\[
\lambda = \frac{340\, \text{m/s}}{68\, \text{Hz}} = 5\, \text{m}
\]

\[
x_1 = 2m + \frac{1}{4}(5m) = 3.25m \quad \text{Answer c)}
\]
8. You are in the front row at a concert, standing 1m away from the speakers. The sound intensity level is an earsplitting 120 db, so you decide to move away to a quieter position. How far away from the speaker do you need to be so that the level is only 80 db?

The formula for intensity level in decibels is

\[
\beta = 10 \cdot \log \left( \frac{I}{I_0} \right)
\]

There is a great shortcut we can use in this problem. We need the level to decrease by 40db, so the Intensity should decrease by a factor of \(10^4\). In general, for every 10db decrease in level, the intensity should be divided by 10 (if the level increases, multiply instead).

The other relationship we need here is

\[
I \propto \frac{1}{r^2}
\]

This says that the Intensity is inversely related to the square of the distance from the source of the sound.

So if the distance increases by a factor of 10, the intensity goes down by a factor of 100.

In this case, we need the intensity to change by a factor of \(10^4\) so the distance should increase by a factor of \(10^2\).

ANSWER: c) 100m
9. A loudspeaker playing a constant frequency tone is dropped off a cliff. As it accelerates downward, a person standing at the bottom of the cliff will hear a sound of:

a) increasing frequency and decreasing amplitude
b) constant frequency and increasing amplitude
c) increasing frequency and increasing amplitude
d) decreasing frequency and constant amplitude

The sound source is getting closer to the person, so it will get louder (increasing amplitude).

Also, the source is **accelerating** toward the listener, so the frequency will be doppler-shifted upward more and more as the speaker speeds up faster and faster.

Answer c)
10. A “boat” consists of a massless hollow cube of side length 50cm, floating in a freshwater lake. When a person steps onto the “boat” it sinks down 30cm. Find the weight of the person.

Before the person steps on, the buoyant force just equals the weight of the boat. After, the buoyant force is the total weight of the boat plus the person.

So the result of the person stepping on is that the buoyant force increases by the weight of the person.

The buoyant force is also the weight of the displaced water. When the person steps on, the extra water displaced has a volume of \((50 \times 50 \times 30) = 75,000 \text{ cm}^3\)

Water has a density of \(1 \text{ g/cm}^3\), so the extra displaced water has a mass of 75kg. This is equal to the mass of the person.

The person’s weight is thus \((75 \text{ kg})(9.8 \text{ m/s}^2) = 735 \text{ N}\)

Round this up to 750N
11. Two strings on the same guitar (same length) are tuned so that string B is one octave higher frequency than string A. Given that string A has 4 times the mass of string B, what is the ratio of the tensions in the strings? (Hint: One octave higher frequency means the frequency is twice as high).

a) String A has 4 times the tension of String B  
b) String A has 2 times the tension of String B  
c) The tensions in the strings are equal  
d) String A has half the tension of String B

The frequencies of the standing waves on a string are given by \( f = \frac{nv}{2L} \). Since the lengths of the strings are the same, we see that the waves on string B must travel at twice the speed of the waves on string A.

Wave speed on a string is given by \( v = \sqrt{\frac{F_{\text{tension}}}{\mu}} \). Solve this for tension to get \( F_{\text{tension}} = v^2 \cdot \mu \). In this case, the mass/length of string A is 4 times as much as string B. The tensions come out to be the same since the double speed (squared) gives a factor of 4 for string B. Answer c).
12. One of the harmonics on a string that is 1.30 m long has a frequency of 15.60 Hz. The next higher harmonic has frequency 23.40 Hz. Find the fundamental frequency and the speed of the waves on the string.

The difference between any 2 successive harmonic frequencies on a particular string is always equal to the fundamental frequency for that string. In this case that is $(23.4 - 15.6) = 7.8$ Hz.

To get the speed, use the formula

$$f_1 = \frac{v}{2L} \Rightarrow v = (7.8 \text{Hz}) (2 \cdot 1.3 \text{m}) = 20.28 \frac{\text{m}}{\text{s}}$$
13. Two protons are placed as follows: Proton A is placed on the X-axis, +5 cm from the origin. Proton B is placed on the Y-axis, +10 cm from the origin. Find the electric field at the origin (magnitude and direction). The charge on a proton is $1.6 \times 10^{-19}$ C.

There are 2 charges, so we calculate 2 contributions to the E-field. The formula will calculate magnitudes, and the direction is away from each charge since they are positive.

$$|\vec{E}| = \frac{kq}{r^2}$$

$$E_A = \left( \frac{9 \times 10^9 \text{ N} \cdot \text{m}^2}{\text{C}^2} \right) \left( 1.6 \times 10^{-19} \text{ C} \right) \left( 0.05 \text{ m} \right)^2 = 5.76 \times 10^{-7} \text{ N/C}$$

$$E_B = \left( \frac{9 \times 10^9 \text{ N} \cdot \text{m}^2}{\text{C}^2} \right) \left( 1.6 \times 10^{-19} \text{ C} \right) \left( 0.1 \text{ m} \right)^2 = 1.44 \times 10^{-7} \text{ N/C}$$

We need to use the Pythagorean Theorem to find the magnitude of the resultant when these 2 vectors are added:

$$E_{\text{total}} = \sqrt{(5.76 \times 10^{-7} \text{ N/C})^2 + (1.44 \times 10^{-7} \text{ N/C})^2} = 5.9 \times 10^{-7} \text{ N/C}$$

To find the angle, we can just look at the diagram (to the left and down means below the negative x-axis), or we can calculate the value using right triangle rules:

$$\tan(\theta) = \frac{E_B}{E_A} \Rightarrow \theta = \tan^{-1}\left( \frac{1.44}{5.76} \right) = 14^\circ$$