Physics 2

Chapter 12 problems
A tall cylinder with a cross-sectional area 12cm$^2$ is partially filled with mercury; the surface of the mercury is 5cm above the bottom of the cylinder. Water is slowly poured in on top of the mercury, and the two fluids don’t mix. What volume of water must be added to double the gauge pressure at the bottom of the cylinder?

\[
\text{Gauge pressure before pouring the water is due to the mercury:} \\
p_{\text{initial}} = \rho_{\text{mercury}} \cdot g \cdot h_{\text{mercury}} \\
p_{\text{initial}} = (13600 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.05 \text{ m}) = 6664 \text{ Pa} \\
\]

\[
\text{After pouring the water, the gauge pressure doubles:} \\
p_{\text{final}} = 2 \cdot 6664 \text{ Pa} = 13328 \text{ Pa} \\
13328 \text{ Pa} = \rho_{\text{water}} \cdot g \cdot h_{\text{water}} + \rho_{\text{mercury}} \cdot g \cdot h_{\text{mercury}} \\
6664 \text{ Pa} \\
\]

\[
\text{So} \\
\rho_{\text{water}} \cdot g \cdot h_{\text{water}} = 6664 \text{ Pa} \\
(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2) \cdot h_{\text{water}} = 6664 \text{ Pa} \\
h_{\text{water}} = 0.68 \text{ m} \\
\]

\[
\text{Volume of water = (area)(height)} \\
V = (12 \text{ cm}^2)(0.68 \text{ cm}) \\
V = 8.16 \text{ cm}^3 \\
\]
A hollow plastic sphere is held below the surface of a freshwater lake by a cord anchored to the bottom of the lake. The sphere has a volume of 0.65 m$^3$ and the tension in the cord is 900 N.

(a) Calculate the buoyant force exerted by the water on the sphere.

(b) What is the mass of the sphere?

(c) The cord breaks and the sphere rises to the surface. When the sphere comes to rest, what fraction of its volume will be submerged?

\[ V = 0.65 \text{ m}^3 \]
\[ T = 900 \text{ N} \]

\[ F_B = \rho_{\text{water}} \cdot g \cdot V_{\text{sub}} \]

\[ F_B - mg - T = 0 \Rightarrow mg = F_B - T = 6300 - 900 = 5400 \text{ N} \]

\[ m = \frac{5400}{9.8} = 552 \text{ kg} \]

\[ F_B - mg = 0 \]

\[ \rho_{\text{water}} \cdot g \cdot V_{\text{sub}} = mg = \rho V \]

\[ V_{\text{sub}} = \frac{m}{\rho_{\text{water}}} = \frac{552}{1000} = 0.552 \text{ m}^3 \]

\[ \frac{V_{\text{sub}}}{V} = \frac{0.552}{0.65} = 0.859 \]

This is also

\[ \frac{\rho_{\text{sphere}}}{\rho_{\text{water}}} = \frac{0.859}{1.000} = 0.859 \]
A rock is suspended by a light string. When the rock is in air, the tension in the string is 39.2N. When the rock is totally submerged in water, the tension is 28.4N. When the rock is totally submerged in an unknown liquid, the tension is 18.6N. What is the density of the unknown liquid?

\[
\begin{align*}
\text{In air, } F_B &= 0 \text{ because } F_{\text{air}} \text{ is tiny} \\
\text{In water, } F_B &= 10.8 \text{ N} \\
\text{In unknown, } F_B &= 20.6 \text{ N}
\end{align*}
\]

\[
\rho_{\text{g. v}} = \frac{F_B}{g} = \begin{cases} 10.8 \text{ N} & \text{in water} \\ 20.6 \text{ N} & \text{in unknown} \end{cases}
\]

\[
\frac{\rho_u}{\rho_w} = \frac{20.6}{10.8} \Rightarrow \rho_u = 1907 \frac{\text{kg}}{\text{m}^3}
\]
At one point in a pipeline, the water’s speed is 3 m/s and the gauge pressure is 50 kPa. Find the gauge pressure at a second point on the line that is 11 m lower than the first if the pipe diameter at the second point is twice that of the first.

We need Bernoulli’s Equation for this one (really it’s just conservation of energy for fluids).

Notice we set up the y-axis so point 2 is at y=0.

Here’s Bernoulli’s equation – we need to find the speed at point 2 using continuity, then plug in the numbers.

\[ p_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2 \]

Continuity Equation:

\[ A_1 \cdot v_1 = A_2 \cdot v_2 \]

This is the ratio of the AREAS – it is the square of the ratio of the diameters

\[ v_2 = \frac{A_1}{A_2} \cdot v_1 \Rightarrow v_2 = \frac{1}{4} \cdot v_1 = 0.75 \text{ m/s} \]

Plugging in the numbers to Bernoulli’s Equation:

\[ p_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2 \]

\[ 50,000 \text{ Pa} + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(11 \text{ m}) + \frac{1}{2} (1000 \text{ kg/m}^3)(3 \text{ m/s})^2 = p_2 + 0 + \frac{1}{2} (1000 \text{ kg/m}^3)(0.75 \text{ m/s})^2 \]

\[ p_2 = 162,000 \text{ Pa} \]
A U-shaped tube open to the air at both ends contains some mercury. A quantity of water is carefully poured into the left arm of the U-shaped tube until the vertical height of the water column is 15cm.

(a) What is the gauge pressure at the water-mercury interface?
(b) Calculate the vertical distance \( h \) from the top of the mercury in the right-hand arm of the tube to the top of the water in the left-hand arm.

(a) Gauge pressure at the interface is due to the water

\[
P_{\text{water}} = \rho g h = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.15 \text{ m}) = 1470 \text{ Pa}
\]

(b) The height of the mercury (above the bottom of the water) is \( (15 \text{ cm} - h) \).

The pressures must be equal at equal depths, so we know the gauge pressure due to the mercury must be the same as the pressure in part of from the water.

\[
P_{\text{mercury}} g (15 \text{ cm} - h) = 1470 \text{ Pa}
\]

\[
(13,600 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.15 \text{ m} - h) = 1470 \text{ Pa}
\]

\[
0.15 \text{ m} - h = 0.011 \text{ m} \implies h = 0.139 \text{ m} = 13.9 \text{ cm}
\]
A hot-air balloon has a volume of 2200m³. The balloon fabric (the envelope) weighs 900N. The basket with gear and full propane tanks weighs 1700N. If the balloon can safely lift and additional 3200N of passengers, breakfast, and champagne when the outside air density is 1.23 kg/m³, what is the average density of the heated gases in the envelope?

\[ P_{\text{Air}} = 1.23 \frac{\text{kg}}{\text{m}^3} \]

\[ \Sigma F = 0 \]

\[ -3200 \text{N} - 1700 \text{N} - 900 \text{N} - W_{\text{Gas}} + \overline{F}_B = 0 \]

\[ -5800 \text{N} - m_{\text{Gas}} g + \rho_{\text{Air}} g = 0 \]

\[ m_{\text{Gas}} = \frac{(1.23 \frac{\text{kg}}{\text{m}^3})(2200 \text{m}^3)(9.8 \frac{\text{m}}{\text{s}^2}) - 5800 \text{N}}{9.8 \frac{\text{m}}{\text{s}^2}} \]

\[ m_{\text{Gas}} = 2114 \text{ kg} \]

\[ P_{\text{Gas}} = \frac{2114 \text{ kg}}{2200 \text{ m}^3} \]

\[ P_{\text{Gas}} = 0.96 \frac{\text{kg}}{\text{m}^2} \]
The horizontal pipe shown has a cross-sectional area of 40 cm² at the wider portions and 10 cm² at the constriction. Water is flowing in the pipe, and the discharge from the pipe is 6x10⁻³ m³/s. Find (a) the flow speeds at the wide and narrow portions; (b) the pressure difference between these portions; (c) the difference in height between the mercury columns in the U-shaped tube.

a) Find flow speeds.

Flow = (Area)(Velocity)

\[ 6 \times 10^{-3} \frac{m^3}{s} = (40 \text{ cm}^2)(V_{\text{wide}}) \]

\[ 6 \times 10^{-3} \frac{m^3}{s} = (4 \times 10^{-4} \text{ m}^2)V_{\text{wide}} \rightarrow V_{\text{wide}} = 1.5 \frac{m}{s} \]

\[ 6 \times 10^{-3} \frac{m^3}{s} = (10 \times 10^{-4} \text{ m}^2)V_{\text{narrow}} \rightarrow V_{\text{narrow}} = 6 \frac{m}{s} \]

b) Use Bernoulli's Equation:

\[ P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2 \]

\[ P_1 - P_2 = \rho g (y_2 - y_1) + \frac{1}{2} \rho (v_2^2 - v_1^2) \]

\[ P_1 - P_2 = \frac{1}{2} (1000 \text{ kg/m}^3)(1.58^2 - 1.58^2) = 16,875 \text{ Pa} \]

c) The pressure difference will match the gauge pressure of the extra column of mercury:

\[ P_{\text{merc}} \cdot s \cdot h = 16,875 \text{ Pa} \]

\[ h = \frac{16,875 \text{ Pa}}{(13,600 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 1.27 \text{ m} \]

\[ h = 12.7 \text{ cm} \]