Circular Motion

Physics 6A
We have a formula that we will use often for circular motion.

For an object moving in a circular path, the centripetal (toward the center) acceleration is given by:

\[ a_{\text{cent}} = \frac{v^2}{R} \]

Here \( v \) stands for the linear speed and \( R \) is the radius of the circular path.

You might also see \( a_{\text{rad}} \), which stands for radial acceleration.

Notice that the radial acceleration is always toward the center of the circle, and the velocity is always tangent to the circle.

This is Uniform Circular Motion.
Ride your bike around a curve and you will notice that if you go too fast, your tires will slip and you will fall. Why does this happen?
Ride your bike around a curve and you will notice that if you go too fast, your tires will slip and you will fall. Why does this happen? Static friction is not strong enough to keep your tires from slipping on the pavement.
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OK, let’s say you are riding your bike around a level curve and your maximum speed is $v$ when the radius of the curve is $R$. Here are a couple of multiple choice questions:

1) What is your maximum speed if the radius of the curve is $2R$?
   - a) $\frac{v}{2}$
   - b) $v$
   - c) $\sqrt{2}v$
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We will need to find a formula relating $v$ and $R$. A diagram may help.
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Notice that the friction force points toward the center of the curve. It is the centripetal force.
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$$\text{friction} = \frac{mv^2}{R}$$

View from above
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   View from above

   Notice that the friction force points toward the center of the curve. It is the centripetal force.

   \[
   \text{friction} = \frac{mv^2}{R}
   \]

   We know a formula for friction as well:

   \[
   \mu_s mg = \frac{m(v_{\text{max}})^2}{R}
   \]

   Maximum static friction will give maximum speed.
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$$v_{max} = \sqrt{\mu_s g R}$$

Solve for $v_{max}$
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   Maximum static friction will give maximum speed.

   If \( R \) is doubled, \( v_{\text{max}} \) increases by \( \sqrt{2} \)
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   b) $v$
   
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   d) $2v$

2) What is your maximum speed if the radius is $R$, but the road is wet, so that your coefficient of static friction is only $1/3$ of the value when the road is dry?
   
   a) $\frac{v}{3}$
   
   b) $\frac{v}{\sqrt{3}}$
   
   c) $\sqrt{3}v$
   
   d) $3v$
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2) What is your maximum speed if the radius is \( R \), but the road is wet, so that your coefficient of static friction is only \( \frac{1}{3} \) of the value when the road is dry?
   
   a) \( \frac{v}{3} \)  
   
   b) \( \frac{v}{\sqrt{3}} \)
   
   c) \( \sqrt{3}v \)
   
   d) \( 3v \)

We can use our formula from part 1)

\[ v_{max} = \sqrt{\mu_s gR} \]
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We can use our formula from part 1)
\[
{v_{\text{max}}} = \sqrt{\mu_s g R}
\]

If \( \mu_s \) decreases to \( \mu_s/3 \) then \( v_{\text{max}} \) will decrease by \( \sqrt{3} \).
Wheel of Doom!

This carnival ride is a giant metal cylinder which will spin around and pin the occupants to the wall. The fun part is when the floor drops out from below and the patrons see a spike-filled pit of angry crocodiles awaiting them should they fall. As safety inspector, your problem will be to determine when it will be unsafe to ride. The given information is this: Radius of cylinder = 20m. Speed of rotation = 10 rpm.

a) Will leather-clad Biker Bob (mass = 100kg ; coeff. of static friction = 0.6) be safe?

b) How about Disco Stu, a 75kg man wearing a silk shirt and polyester pants (\(\mu_s=0.15\))?
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We can start by drawing a free-body diagram of the forces on the person.
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The normal force is the force of the wall pushing inward. This is a centripetal force (it points toward the center of the circle).
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The normal force is the force of the wall pushing inward. This is a centripetal force (it points toward the center of the circle).

We can write down our formula for centripetal force:

\[ \sum F_{\text{cent}} = \frac{mv^2}{R} \]

\[ N = \frac{mv^2}{R} \]
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The vertical forces must balance out if the person wants to avoid the crocodile pit, so we can write down a formula:

\[ \text{friction} = mg \]

What type of friction do we want – static or kinetic?
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\[
\text{friction} = mg \\
\mu_s \cdot N = mg
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By putting the maximum force of static friction in our formula, we are assuming the man is just on the verge of sliding.
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friction = mg
μ_s · N = mg
μ_s · \frac{mv^2}{R} = mg

We can replace N with the expression we found earlier.
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\mu_s \cdot \frac{mv^2}{R} = mg \\
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Now that we have this formula, how do we use it?

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\[ \mu_s \cdot \frac{mv^2}{R} = mg \]
\[ \mu_s \cdot \frac{v^2}{R} = g \]
\[ \mu_s = \frac{gR}{v^2} \]

Notice that the mass canceled out, so based on the given information we should solve for \( \mu \).
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\[
\begin{align*}
\text{friction} &= mg \\
\mu_s \cdot N &= mg \\
\mu_s \cdot \frac{mv^2}{R} &= mg \\
\mu_s \cdot \frac{v^2}{R} &= g \\
\mu_s &= \frac{gR}{v^2}
\end{align*}
\]

The radius and speed are given, but the speed is in rpm, so we will need to convert it to m/s.
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\[
\frac{10 \text{ rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{2\pi \cdot 20 \text{ m}}{\text{rev}} \approx 21 \frac{\text{m}}{\text{s}}
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$\mu_s = \frac{gR}{v^2}$

Substitute the values for g, R and v

$$\mu_s = \frac{(9.8 \text{ m/s}^2)(20\text{m})}{(21\text{m/s})^2} = 0.44$$
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\mu_s \cdot \frac{v^2}{R} = g \\
\mu_s = \frac{gR}{v^2} \\
\mu_s = \left( \frac{9.8 \text{ m/s}^2}{21 \text{ m/s}} \right)^2 = 0.44 \\
\]

So if the coefficient is 0.44 the person will be on the verge of sliding down into the pit.
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Biker Bob is safe (his 0.6 coefficient is larger than 0.44, so static friction is enough to hold him in place)

Disco Stu is doomed! (his 0.15 coefficient is too small, so static friction fails to hold him in place)