Multiple-choice questions (56 points; 8 points per question). For each of these seven questions, mark a clear “X” in the box next to the one best answer.

1. A mass on a spring oscillates with amplitude $A$ and period $T$. How long does it take the mass to move from $x = A$ to $x = 0.500A$?

- A. $0.0625T$
- B. $0.0833T$
- C. $0.125T$
- D. $0.167T$
- E. $0.250T$
- F. $0.500T$
- G. $0.524T$

$x = A \cos \left( \omega t + \phi \right) = A \cos \left( \frac{2\pi}{T} t + \phi \right)$

For simplicity, let $\phi = 0$, so $x = A \cos \left( \frac{2\pi}{T} t \right)$.

Then $x = A$ at $t = 0$

$x = 0.500 A = A \cos \left( \frac{2\pi}{T} t \right)$ for what $t$?

$\cos \left( \frac{2\pi}{T} t \right) = 0.500$, $\frac{2\pi}{T} t = \cos^{-1}(0.500)$ in radians

$t = T \cdot \frac{\cos^{-1}(0.500)}{2\pi} = 0.167 T$

2. A substance has a melting point of $20^\circ C$ and a heat of fusion of $3.5 \times 10^4$ J/kg. The boiling point is $150^\circ C$ and the heat of vaporization is $7.0 \times 10^4$ J/kg at a pressure of 1.0 atm. The specific heats for the solid, liquid, and gaseous phases are $600$ J/(kg·K), $1000$ J/(kg·K), and $400$ J/(kg·K), respectively. The quantity of heat given up by $0.50$ kg of the substance when it is cooled from $170^\circ C$ to $88^\circ C$, at a pressure of 1.0 atmosphere, is closest to

- A. 14 kJ.
- B. 21 kJ.
- C. 28 kJ.
- D. 30 kJ.
- E. 44 kJ.
- F. 60 kJ
- G. 70 kJ.
- H. 88 kJ
- I. 140 kJ.

$170^\circ C \rightarrow 88^\circ C$ : substance goes from gas to liquid.

Heat given up by substance when $T_{gass} 170^\circ C \rightarrow 88^\circ C$

equals heat into substance when $T_{gass} 88^\circ C \rightarrow 170^\circ C$

This is

$Q = m C_{\text{liquid}} (150^\circ C - 88^\circ C)$

$+ m C_{\text{vaporization}}$

$+ m C_{\text{gas}} (170^\circ C - 150^\circ C)$

$\text{with } m = 0.50 \text{ kg}, C_{\text{liquid}} = 1000 \frac{J}{\text{kg} \cdot \text{K}}, C_{\text{vaporization}} = 7.0 \times 10^4 \frac{J}{\text{kg}}, C_{\text{gas}} = 400 \frac{J}{\text{kg} \cdot \text{K}}$, get

$Q = 7.0 \times 10^4 J = 70 \times 10^3 J = 70 \text{ kJ}$

(MULTIPLE-CHOICE QUESTIONS CONTINUE ON THE NEXT PAGE)
3. A lightly damped harmonic oscillator, with a damping force proportional to its speed, is oscillating with an amplitude of 0.500 cm at time $t = 0$. When $t = 8.20\, s$, the amplitude has died down to 0.400 cm. At what value of $t$ will the oscillations have an amplitude of 0.250 cm?

- A. 5.13 s
- B. 12.4 s
- C. 16.5 s
- D. 18.5 s
- E. 20.5 s
- F. 25.5 s

At $t = 0$, amplitude $= Ae^0 = A = 0.500\, \text{cm}$

At $t_1 = 8.20\, s$, amplitude $= Ae^{-bt_1/2m} = 0.400\, \text{cm}$, so

$e^{-bt_1/2m} = \frac{0.400\, \text{cm}}{0.500\, \text{cm}} = 0.800$ and $-\frac{bt_1}{2m} = \ln(0.800)$, so

$e^{-bt_2/2m} = \frac{0.250\, \text{cm}}{0.500\, \text{cm}} = 0.500$

Time $t_2$ when amplitude $= 0.250\, \text{cm} = (0.500\, \text{cm})e^{-bt_2/2m} = e^{-bt_2/2m}$.

$-\frac{bt_2}{2m} = \ln(0.500)$

$t_2 = -\frac{2m}{b} \ln(0.500)

= -\frac{\ln(0.500)}{(b/2m)}

= -\frac{(-0.693)}{(0.0272\, \text{s}^{-1})}

= 25.5\, \text{s}$

4. An architect is interested in estimating the rate of heat loss through a sheet of insulating material as a function of the thickness of the sheet. Assuming fixed temperatures on the two faces of the sheet, which one of the graphs in the figure best represents the rate of heat loss as a function of the thickness of the insulating sheet?

- A. Graph A
- B. Graph B
- C. Graph C
- D. Graph D
- E. Graph E

So $\frac{bT}{2m} = \ln(0.500)$

$t_2 = -\frac{2m}{b} \ln(0.500)$

$= -\frac{\ln(0.500)}{(b/2m)}$

$= -\frac{(-0.693)}{(0.0272\, \text{s}^{-1})}$

$= 25.5\, \text{s}$

$H = \frac{\Delta Q}{\Delta T} = kA \left( \frac{T_H - T_C}{L} \right)$

Inversely proportional to thickness $L$

(MULTIPLE-CHOICE QUESTIONS CONTINUE ON THE NEXT PAGE)
5. This is a graph of acceleration versus time for an object in simple harmonic motion. At which of the following times does the object have a negative displacement and a positive velocity?

- A. 0.100 s
- B. 0.125 s
- C. 0.150 s
- D. 0.175 s
- E. 0.200 s
- F. 0.225 s
- G. 0.250 s
- H. 0.275 s

In SHM, \( a_x = -\omega^2 x \), so \( x = -\frac{1}{\omega^2} a_x \)

Graph of \( x \) vs. \( t \) is the negative of the graph of \( a_x \) vs. \( t \)

We want a point where \( x < 0 \) (and so \( a_x > 0 \))

but the \( x-t \) graph has a positive slope \( \frac{dx}{dt} = v_x \).

6. A 1.7-kg solid sphere, made of metal whose density is 4400 kg/m³, is suspended by a cord. When the sphere is completely immersed in water (of density 1000 kg/m³), what is the tension in the cord?

- A. 10 N
- B. 11 N
- C. 13 N
- D. 15 N
- E. 17 N
- F. 19 N
- G. 20 N

Not force on sphere = 0

\[ T + B - m_{\text{sphere}} g = 0 \]

so \( T = m_{\text{sphere}} g - B \)

\( m_{\text{sphere}} g = (1.7 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2}) \)

\( B = \text{buoyant force} = \rho_{\text{water}} V_{\text{sphere}} g \).

The volume of the sphere is \( V_{\text{sphere}} = \frac{m_{\text{sphere}}}{\rho_{\text{metal}}} \)

so \( B = \rho_{\text{water}} \frac{m_{\text{sphere}} g}{\rho_{\text{metal}}} = \frac{(1000 \text{ kg/m}^3)(1.7 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2})}{(4400 \text{ kg/m}^3)} \)

\( T = m_{\text{sphere}} g - B = (1.7 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2}) - \frac{(1000 \text{ kg/m}^3)(1.7 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2})}{(4400 \text{ kg/m}^3)} \)

\( = 13 \text{ N} \)
7. A brass rod is 40.1 cm long and an aluminum rod is 79.3 cm long when both rods are at an initial temperature of 0°C. The rods are placed in line with a gap of 0.60 cm between them, as shown in the figure. The distance between the far ends of the rods is maintained at 120.0 cm throughout. The temperature of both rods is raised until the two rods are barely in contact. The coefficients of linear expansion of brass and aluminum are 2.0 \times 10^{-5} \text{ K}^{-1} and 2.4 \times 10^{-5} \text{ K}^{-1}, respectively. At what temperature does contact of the rods barely occur?

- A. 190°C
- B. 200°C
- C. 210°C
- D. 220°C
- E. 230°C
- F. 240°C

Want the combined length increase of the two rods to be 0.6 cm: so

\[ \Delta L_{\text{brass}} + \Delta L_{\text{Al}} = 0.6 \text{ cm} = \Delta L_{\text{total}} \]

\[ \alpha_{\text{brass}} L_{\text{brass}} (T - T_0) + \alpha_{\text{Al}} L_{\text{Al}} (T - T_0) = 0.6 \text{ cm} = \Delta L_{\text{total}} \]

So

\[ \left( \alpha_{\text{brass}} L_{\text{brass}} + \alpha_{\text{Al}} L_{\text{Al}} \right) T = \left( \alpha_{\text{brass}} L_{\text{brass}} + \alpha_{\text{Al}} L_{\text{Al}} \right) T_0 + \Delta L_{\text{total}} \]

\[ T = T_0 + \frac{\Delta L_{\text{total}}}{\left( \alpha_{\text{brass}} L_{\text{brass}} + \alpha_{\text{Al}} L_{\text{Al}} \right)} \]

\[ = 0°C + \frac{0.6 \text{ cm}}{\left(2.0 \times 10^{-5} \text{ K}^{-1}\right)(40.1 \text{ cm}) + \left(2.4 \times 10^{-5} \text{ K}^{-1}\right)(79.3 \text{ cm})} \]

\[ = 0°C + 220 \text{ K} \]

\[ = 220°C \]
Problem (44 points)

The horizontal pipe shown in the figure has radius $r_1$ at the wide portion and radius $r_2$ at the narrow portion. Water is flowing in the pipe at a volume flow rate (in m³/s) equal to $Q$. The density of water is $\rho_w$ and the density of mercury is $\rho_m$.

(a) Find the difference between the pressure at the wide portion of the pipe and the pressure at the narrow portion of the pipe. Your answer should involve no quantities other than $r_1$, $r_2$, $Q$, $\rho_w$, $\rho_m$, and $g$. (It may or may not involve all of these quantities. You will lose points if you substitute a numerical value for $g$.)

For full credit, show your work, simplify your answer, and draw a box around your final answer. You must also state whether the pressure is higher in the wide portion or the narrow portion.

Wide = 1, Narrow = 2

Bernoulli:

$$P_1 + \frac{1}{2} \rho_w v_1^2 + \rho_w g y_1 = P_2 + \frac{1}{2} \rho_w v_2^2 + \rho_w g y_2$$

Points 1 & 2 are at the same height, so $y_1 = y_2$

So

$$P_1 + \frac{1}{2} \rho_w v_1^2 = P_2 + \frac{1}{2} \rho_w v_2^2$$

Continuity:

$$Q = A_1 v_1 = A_2 v_2$$

or

$$Q = \pi r_1^2 v_1 = \pi r_2^2 v_2$$

So

$$v_1 = \frac{Q}{\pi r_1^2}, \quad v_2 = \frac{Q}{\pi r_2^2}$$

So pressure difference is

$$P_1 - P_2 = \frac{1}{2} \rho_w \left[ \left( \frac{Q}{\pi r_2^2} \right)^2 - \left( \frac{Q}{\pi r_1^2} \right)^2 \right]$$

(continued on next page)

\[ P_1 - P_2 = \frac{\rho_w Q^2}{2 \pi^2} \left[ \frac{1}{r_2^4} - \frac{1}{r_1^4} \right] \]

Since $r_2 < r_1, \frac{1}{r_2} > \frac{1}{r_1}$ and $P_1 - P_2$ pressure is higher in the wide portion
(b) Find the difference in height between the mercury columns on the two sides of the U-shaped tube. Your answer should involve no quantities other than \( r_1, r_2, Q, \rho_w, \rho_M, \) and \( g. \) (It may or may not involve all of these quantities. You will lose points if you substitute a numerical value for \( g. \)) For full credit, show your work, simplify your answer, and draw a box around your final answer.

1. Pressure at point 3 is greater than at point 4 by the same amount that pressure at point 1 is greater than at point 2. So

\[
P_3 - P_4 = P_1 - P_2 = \frac{\rho w Q^2}{2\pi^2} \left[ \frac{1}{r_2^4} - \frac{1}{r_1^4} \right]
\]

2. Pressure at points 5 & 6 is the same (both are at the same level in the same stationary fluid, mercury). Now,

\[
P_5 = P_3 + \rho w gh \quad \text{and} \quad P_6 = P_4 + \rho M gh
\]

so

\[
P_3 + \rho w gh = P_4 + \rho M gh
\]

\[
P_3 - P_4 = \rho w gh - \rho w gh = (\rho_M - \rho_w) gh
\]

So

\[
h = \frac{P_3 - P_4}{(\rho_M - \rho_w) g} = \frac{P_1 - P_2}{(\rho_M - \rho_w) g} = \frac{\rho w Q^2}{2\pi^2(\rho_M - \rho_w) g} \left[ \frac{1}{r_2^4} - \frac{1}{r_1^4} \right]
\]

END OF THE EXAM