1. A constant force \( F \) is applied horizontally to a mass \( m \), initially at rest on a frictionless surface. Find the time it takes to move the object a distance \( d \).

2. A constant force \( F \) is applied horizontally to two different masses, one with mass \( M \) and the other with mass \( 2M \). Both are initially at rest.
   a) If the force is applied to each mass over the same distance, which one will have the greater kinetic energy? Which one will have the greater momentum?
   b) If the force is applied to each mass for the same amount of time, which one will have the greater kinetic energy? Which one will have the greater momentum?

3. A golf ball of mass 46 grams is struck at an angle of 45 degrees with the horizontal. The drive lands 200 meters away on a flat fairway. If the golf club and ball are in contact for 7 milliseconds, what is the average force of impact?

4. A 12 gram bullet is fired horizontally into a 100 gram wooden block that is initially at rest on a rough horizontal surface and connected to a spring with a force constant of 150 N/m. If the bullet-block system compresses the spring by 0.8 meters, what was the speed of the bullet just as it enters the block? Assume that the coefficient of kinetic friction between block and surface is 0.6.

5. Find the center of mass of a right triangle of base length \( a \) and height \( b \). Use the result of this to compute the location of the center of mass of the triangle with corners at \((0,0)\), \((1,1)\), and \((3,0)\).

6. A 1kg mass moving to the right at a speed of 2 m/s collides with a 0.5 kg mass moving to the left at 3 m/s. Determine the range of possible final velocities for each mass. Could either mass stop after the collision? If so, which one?
7. Consider a rod of length $L$ and mass $m_1$, with a sphere of radius $R$ and mass $m_2$ affixed to one end. The system is allowed to rotate about a hinge on the left end of the rod, as in the figure.

a) Find the total moment of inertia for this system rotating about the hinge (The moment of inertia for a rod rotating about one end is $\frac{1}{3}ML^2$, and the moment of inertia for a sphere spinning about its axis is $\frac{2}{5}MR^2$.)

b) Find the location of the center of mass of the system, relative to the point of rotation.

c) If the system is released from rest from the horizontal position, find the angular velocity at the bottom.

8. A car accelerates uniformly from rest with an acceleration of 3 meters per second squared. If the tires have a diameter of 0.5 meters, find:

a) The speed of the car after 10 seconds.

b) The distance the car has covered in 10 seconds.

c) The angular acceleration of the tires.

d) The angular velocity of the tires after 10 seconds.

e) The speed of the top of each tire relative to the ground after 10 seconds.

f) The speed of the bottom of each tire relative to the ground after 10 seconds.

g) If the car slams on the brakes after 10 seconds and stops after traveling an additional 20 meters, determine the time required to stop.

9. The system in the figure is accelerating as shown. $m_1 > m_2$, the pulley has a moment of inertia $I$ and radius $r$, and the coefficient of kinetic friction between the second mass and the ramp is $\mu_k$.

a) Compute the acceleration of the system.

b) Compute the tension in each section of the connecting string. Why should they be different?
Physics | Practice Final Solutions

1.) Newton's Law: \( F = ma \Rightarrow a = \frac{F}{m} \)

For constant acceleration, kinematics gives:

\[
d = x - x_0 = v_0 t + \frac{at^2}{2}
\]

Combine

\[
d = \frac{F}{2m}t^2
\]

\[
t = \sqrt{\frac{2md}{F}}
\]

By Work/Energy and Impulse/Momentum

\[
W_{Total} = \Delta K \Rightarrow F \cdot d = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2
\]

Thus

\[
v = \sqrt{\frac{2Fd}{m}}
\]

Impulse = \( \vec{J} = Ft = \Delta \vec{p} = mV - mV_0 \)

\[
Ft = m \sqrt{\frac{2Fd}{m}}
\]

\[
t = \sqrt{\frac{2md}{F}}
\]
2. One dimension motion, relationship between kinetic energy \( K \) and momentum \( p \).
\[
K = \frac{1}{2}mv^2 \quad p = mv \quad v = \frac{p}{m}
\]
Thus \( K = \frac{p^2}{2m} \quad p = \sqrt{2mK} \)

a) IF \( F \) is applied for the same distance \( d \).

Work = \( Fd = \Delta K = K_f - K_i = K_f \)
Both \( M \) and \( 2M \) have the same kinetic energy.

But since \( p = \sqrt{2mK} \), the mass with greater mass receives more momentum, since their \( K \) is the same.

b) IF \( F \) is applied for the same time \( t \).

\[
T = Ft = \Delta p = p_f - p_i = mv_f - mv_i
\]
Thus \( Ft = p_f \)
Both \( M \) and \( 2M \) attain the same momentum.

But since \( K = \frac{p^2}{2m} \), the smaller mass attains a greater kinetic energy.
3.) Apply equations of projectile motion.

Apply y displacement equation for returning to the initial height

\[ 0 = Voy t - \frac{g}{2} t^2 = t(Voy - \frac{g}{2} t) \]

Solutions \( t = 0 \), \( t = \frac{2 Voy}{g} \)

So distance traveled in x direction gives

\[ 200 m = Vox t = \frac{2 \cdot Voy \cdot Vox}{g} \]

Since launched at 45°, \( \sin 45° = \frac{\sqrt{2}}{2} \), \( \cos 45° = \frac{\sqrt{2}}{2} \)

The initial velocity relationships are

\[ Vox = Vo \cos 45° = Vo \cdot \frac{\sqrt{2}}{2} \]
\[ Voy = Vo \sin 45° = Vo \cdot \frac{\sqrt{2}}{2} \]

Thus \( 200 = \frac{2}{g} \cdot Vo^2 \left( \frac{\sqrt{2}}{2} \right)^2 = \frac{Vo^2}{g} \)

\[ Vo = \sqrt{200 m \times 9.8 m/s^2} = 44.27 m/s \]

Impulse / momentum \( F \Delta t = p = mV_o \)

\[ F = \frac{mV_o}{\Delta t} = \frac{0.46 \text{ kg} \times 44.27 \text{ m/s}}{0.0075} \]

\[ F = 291 \text{ N} \]
4) This problem consists of a collision of a bullet with a wood block in which momentum is conserved, followed by a sliding of the block during which a spring is compressed and friction work is done.

Since we know everything about the spring compression and friction work, we can calculate the kinetic energy of the combined block and bullet immediately after the collision.

\[ W_f = E_2 - E_1 \]

\[ -\mu_k m_T g x = \frac{1}{2} k x^2 - \frac{1}{2} m_T v_1^2 \quad (x = \text{spring compression}) \]

\[ v_1 = \sqrt{\frac{2 \left( \frac{kx^2}{2} + \mu_k m_T g x \right)}{m_T}} = 29.44 \text{ m/s} \]

Since \( F_{\text{ext}} = 0 \) during collision, \( \Delta p_x = 0 \)

\[ m_b v_b + m_w v_w = m_T v_1 \]

\[ v_b = \frac{m_T v_1}{m_b} = 2.75 \text{ m/s} \]
4) Problem consists of a completely inelastic collision followed by motion with friction and spring compression described by the work/energy equation. Let:

1. Be the condition of bullet moving and block at rest with spring not compressed.
2. Be the condition immediately after a completely inelastic collision with the bullet and wood block having a common velocity but the combination has not yet moved.
3. Be the condition that combined bullet/block has moved, with friction doing negative work, and is now at rest and maximum compression.

\[ x = \text{spring compression}, \quad k = \text{spring constant} \]
\[ \mu_k = \text{coefficient of kinetic friction} \]
\[ m_b = \text{mass of bullet}, \quad m_w = \text{mass of wood block} \]
\[ m_T = \text{combined mass of bullet and wood block} \]
\[ V_b = \text{initial speed of bullet} \]
\[ V_2 = \text{common wood block/bullet speed after collision} \]

\[ E_3 = \frac{1}{2} k x^2 \quad E_2 = \frac{1}{2} m_T V_2^2 \]

\[ W_{other} = -\mu_k m_T g x \]

\[ W_{other} = E_3 - E_2 \quad \Rightarrow \quad V_2 = 29.4 \text{ m/s} \]

1. \[ \rightarrow \] 2. \[ \text{force} = 0 \text{ in horizontal direction} \]

\[ \text{Thus } \Delta p = 0 \text{ in horizontal direction} \]
\[ m_b V_b + m_w V_w^o = m_T V_2 \]

\[ \Rightarrow \quad V_b = 2.75 \text{ m/s} \]
By Math 3B, with \( \rho = \text{constant} \)

\[
X_{cm} = \frac{\int_0^a x \, b \, x \, dx}{\int_0^a b \, x \, dx} = \frac{\frac{a^2 b}{3}}{\frac{a b}{2}} = \frac{2}{3} a
\]

\[
y_{cm} = \frac{\int_0^a \frac{b^2 x^2}{2} \, dx}{\int_0^a b \, x \, dx} = \frac{\frac{a b^2}{6}}{\frac{ab}{2}} = \frac{b}{3}
\]

Interpret this result for the center of mass of any right triangle. The cm is located \( \frac{1}{3} \) of the respective side length from the corner with the right angle.

For desired

\[
\begin{align*}
\text{Triangle 1: } & (x_1, y_1) = \left( \frac{2}{3}, \frac{1}{3} \right) & m_1 = A_1 = \frac{1}{2} \\
\text{Triangle 2: } & (x_2, y_2) = \left( \frac{5}{3}, \frac{1}{3} \right) & m_2 = A_2 = 1 \\
X_{cm} &= \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} = \frac{\frac{2}{3} \times \frac{1}{2} + \frac{5}{3} \times 1}{\frac{1}{2} + 1} = \frac{4}{3} \\
y_{cm} &= \frac{y_1 m_1 + y_2 m_2}{m_1 + m_2} = \frac{\frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times 1}{\frac{1}{2} + 1} = \frac{1}{3}
\end{align*}
\]

Center of mass of object at \( \left( \frac{4}{3}, \frac{1}{3} \right) \)
The extremes of possible collisions are elastic and completely inelastic. In elastic, momentum is conserved and energy is conserved. In completely inelastic, momentum is conserved but the maximum of energy is lost as the two masses have a common velocity after collision. Calculate these two cases for the before collision situation:

\[
\begin{array}{ccc}
\text{m}_1 = 1 \text{kg} & \text{m}_2 = 0.5 \text{kg} \\
\text{2 m/s} & \text{3 m/s} & + x
\end{array}
\]

**Completely inelastic:** \( \Delta p = 0 \)

\[
m_1 V_{1i} + m_2 V_{2i} = (m_1 + m_2) V_{12f}
\]

\[
1 \times 2 - \frac{1}{2} \times 3 = (1 + \frac{1}{2}) V_{12f} \Rightarrow V_{12f} = \frac{1}{3} \text{ m/s}
\]

**Elastic:** \( \Delta p = 0, \Delta E = 0 \)

\[
\Delta p = 0 \Rightarrow m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f} \quad (1)
\]

\[
\Delta E = 0 \Rightarrow \Delta K = 0 \Rightarrow \frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2 \quad (2)
\]

Equations (1) and (2) may be algebraically combined to give still equation (1) but an easier equation instead of (2), that equation is

\[
V_{1i} - V_{2i} = -(V_{1f} - V_{2f}) \quad (3)
\]

Thus (1) gives

\[
1 \times 2 - 0.5 \times 3 = V_{1f} + \frac{1}{2} V_{2f}
\]

or \( V_{1f} + \frac{1}{2} V_{2f} = \frac{1}{2} \)

and (3) gives \( V_{1f} - V_{2f} = -5 \)

Solve obtaining \( V_{1f} = -\frac{11}{3}, \quad V_{2f} = \frac{11}{3} \)

So \( m_1: \quad -\frac{11}{3} \text{ m/s} \leq V_{1f} \leq \frac{11}{3} \text{ m/s} \)

\( m_2: \quad \frac{11}{3} \text{ m/s} \leq V_{2f} \leq \frac{11}{3} \text{ m/s} \)

For some inelastic collision, \( m_1 \) could stop, \( m_2 \) never stop.
7.) For a system of objects, the moment of inertia of the system is the sum of the individual object moment of inertias, all calculated with respect to a single axis. The moment of inertia with respect to an axis parallel to an axis through the object's center of mass is

\[ I = I_{cm} + Md^2, \quad d = \text{distance between axes} \]

a) \[ I_{total} = \frac{1}{3} m_1 L^2 + \frac{2}{5} m_2 R^2 + m_2 (L+R)^2 \]

This is with respect to an axis through the pivot point and \( \perp \) to page.

b) With \( x=0 \) at the pivot

\[ x_{cm} = \frac{m_1 \times \frac{L}{2} + m_2 (L+R)}{m_1 + m_2} \]

c.) \( \omega = 0 \) (Forces at pivot do no work)

\[ \Delta E = 0, \quad E_f = E_i \]

Choose \( y = 0 \) based on origin located distance \( x_{cm} \) below the pivot.

\[ E_f = K_f = \frac{1}{2} I_{total} \omega^2, \quad E_i = (m_1 + m_2) g x_{cm} \]

The axis of rotation has no speed so no linear kinetic energy.

Thus

\[ \omega = \sqrt{\frac{2 (m_1 + m_2) g x_{cm}}{I_{total}}} \]
8.) Linear and Rotational Kinematics

\[ a = \text{Constant} \quad \alpha = \text{Constant} \]
\[ V = V_0 + at \quad \omega = \omega_0 + \alpha t \]
\[ x = x_0 + V_0 t + \frac{at^2}{2} \quad \theta = \theta_0 + \omega_0 t + \frac{\alpha t^2}{2} \]
\[ v^2 = v_0^2 + 2a(x-x_0) \quad \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \]
\[ x = x_0 + \left(\frac{V+V_0}{2}\right)t \quad \theta = \theta_0 + \left(\frac{\omega+\omega_0}{2}\right)t \]

a) \[ V = V_0 + at \quad V = 0 + 3m/s^2 \times 10s = 30m/s \]

b) \[ x = x_0 + V_0 t + \frac{at^2}{2} \quad x = 3m/s^2 \times (10s)^2 = 150m \]

c) \[ a_T = \alpha r \quad \alpha = \frac{a_T}{r} = \frac{3m/s^2}{2.5m} = 12 \text{ rad/s}^2 \]

d) \[ v_T = \omega r \quad \omega = \frac{v_T}{r} = \frac{30m/s}{2.5m} = 120 \text{ rad/s} \]

Note: Center of wheel has \( v = 30m/s \), \( a = 3m/s^2 \)
and contact point with ground has \( v = 0 \)

e) Top of wheel moves twice as fast as center of wheel \( v_{\text{Top}} = 60m/s \)

f) Bottom of tire is always instantaneously stopped (unless tire is slipping)

\[ v_{\text{Bottom}} = 0 \]

g) Car stops from \( V = 30m/s \) in distance 20m.
(\text{Constant acceleration is assumed.})

\[ x-x_0 = \left(\frac{V_0+V}{2}\right)t \]
\[ \Rightarrow \quad t = \frac{2(x-x_0)}{V_0+V} = \frac{2\times(20m-0m)}{30m/s+0} = \frac{4}{3} \text{ s} \]
9.) Base analysis on free body diagrams of:

\[ m_1 \]
\[ a \downarrow \]
\[ T_1 \]
\[ \omega_1 = m_1 g \]

\[ m_1 g - T_1 = m_1 a \quad \text{(1)} \]

Pulley

\[ \alpha \]
\[ T_2 \]
\[ \text{No slip of cable} \]
\[ \text{Thus } v_T = r \omega \]
\[ a = \omega r \]
\[ \alpha = \frac{a}{r} \]

\[ T_1 - T_2 = I \alpha \]
\[ \frac{T_1 r - T_2 r}{r} = I \alpha \quad \Rightarrow \quad T_1 - T_2 = \frac{I a}{r^2} \quad \text{(2)} \]

\[ m_2 \]
\[ T_2 \]
\[ f_k \]
\[ a \]
\[ y: n - m_2 g \cos \theta = 0 \]
\[ n = m_2 g \cos \theta \]
\[ f_k = \mu_k n = \mu_k m_2 g \cos \theta \]

\[ x: T_2 - m_2 g \sin \theta - f_k = m_2 a \]
\[ T_2 - m_2 g \sin \theta - \mu_k m_2 g \cos \theta = m_2 a \quad \text{(3)} \]

Add equations (1), (2) and (3)

\[ m_1 g - m_2 g \sin \theta - \mu_k m_2 g \cos \theta = (m_1 + m_2 + \frac{I}{r^2})a \]

\[ a = \frac{g(m_1 - m_2 \sin \theta - \mu_k m_2 \cos \theta)}{m_1 + m_2 + \frac{I}{r^2}} \]

\[ T_1 = m_1 (g - a) \]
\[ T_2 = m_2 (a + g (\sin \theta + \mu_k \cos \theta)) \]

\( T_1 \) and \( T_2 \) are different because a net torque is required to accelerate the pulley.
Base analysis on Work/Energy

To simplify, assume system starts from rest and the vertical (positive up) position of both masses is initially zero. Let $1$ be mass $1$, $2$ be mass $2$, $p$ be pulley. Let $i$ be the initial state and $f$ be the final state after $m_1$ has dropped a distance $d$.

By these assumptions $E_i = 0$

$E_f = K_{1f} + K_{2f} + K_{pf} + U_{gf1} + U_{gf2}$

$V =$ speed of both $m_1$ and $m_2$ at final condition

By no slip at the pulley, $\omega = \frac{V}{r}$

$E_f = \frac{1}{2} m_1 V^2 + \frac{1}{2} m_2 V^2 + \frac{1}{2} \frac{I}{r^2} V^2 + m_1 g (-d) + m_2 g d \sin \theta$

$W_{other} = W_{friction} = -f_k d = -\mu_k m_2 g \cos \theta d$

$W_{other} = E_f - E_i$

$V^2 = \frac{2 g d (m_1 - m_2 \sin \theta - \mu_k m_2 \cos \theta)}{m_1 + m_2 + \frac{I}{r^2}}$

By kinematics $V^2 = V_0^2 + 2ad \Rightarrow a = \frac{V^2}{2d}$

So $a = \frac{g (m_1 - m_2 \sin \theta - \mu_k m_2 \cos \theta)}{m_1 + m_2 + \frac{I}{r^2}}$

$T_1$ and $T_2$ follow from free body analysis of $m_1$ and $m_2$ as previous.