1. (This problem is worth 40 points.) A roller coaster car of mass $m$ travels around a vertical loop of radius $R$. There is no friction and no air resistance. At the top of the loop (position 1 in the diagram) the downward normal force on the car has magnitude $mg$.

(a) In the space below, draw the free-body diagram for the car at position 1 and the free-body diagram for the car at position 2. For each force in your diagrams, indicate what object exerts that force.

(b) Find the net force on the car at position 2. Give both the magnitude of the net force and the direction of the net force vector relative to the horizontal. Include a drawing that shows the direction of the net force at position 2. Your answer should involve no quantities other than $m$, $R$, and $g$. (It may or may not involve all of these.) For full credit, show your work, simplify your answer, and draw a box around your answer.

(c) Now suppose there is friction between the car and the track, and the friction force has a constant magnitude. In this case the speed of the car when it reaches position 3 (at the bottom of the loop) is $3/2$ times the speed at position 1. Find the magnitude of the friction force. Your answer should involve no quantities other than $m$, $R$, and $g$. (It may or may not involve all of these.) For full credit, show your work, simplify your answer, and draw a box around your answer.
2. (This problem is worth 40 points.)
Two blocks, one of mass $m_1$ and one of mass $m_2$, rest on an inclined ramp of angle $\beta$ as shown. The blocks are connected by a massless rope that does not stretch. There is no friction between the ramp and the block of mass $m_1$, but there is friction (coefficient of kinetic friction $\mu_k$) between the ramp and the block of mass $m_2$. When released from rest, the two blocks accelerate together down the ramp.

(a) In the space to the right, draw the free-body diagram for the block of mass $m_1$.
For each force in your diagram, indicate what object exerts that force.

(b) In the space to the right, draw the free-body diagram for the block of mass $m_2$.
For each force in your diagram, indicate what object exerts that force.

(c) Find the tension in the rope as the blocks slide down the ramp. Your answer should involve no quantities other than $\mu_k$, $m_1$, $m_2$, $g$, and $\beta$. (It may or may not involve all of these.) For full credit, show your work, simplify your answer, and draw a box around your answer.
3. MULTIPLE CHOICE. (This problem is worth 20 points.)
For each question, draw a circle around the one best answer. Each question is worth 4 points. There is no penalty for guessing.

(a) A lightweight crate (A) and a heavy crate (B) are side-by-side on a frictionless horizontal surface. If you apply a horizontal force $F$ to crate A as shown,

(i) the acceleration is greater than if B were on the left and A were on the right
(ii) the acceleration is less than if B were on the left and A were on the right
(iii) the crates will not move if $F$ is less than the combined weight of A and B
(iv) both (i) and (ii) are correct, but not (iii)
(v) both (i) and (iii) are correct, but not (ii)
(vi) both (ii) and (iii) are correct, but not (i)
(vii) all of (i), (ii), and (iii) are correct
(viii) none of (i), (ii), or (iii) are correct

(b) A traffic light of weight $w$ hangs from two lightweight cables, one on each side of the light. Each cable hangs at a 45° angle from the horizontal. What is the tension in each cable?

(i) $w/2$
(ii) $w/\sqrt{2}$
(iii) $w$
(iv) $w\sqrt{2}$
(v) $2w$

(c) A nonzero net force acts on an object. Which of the following quantities could be constant?

(i) the object’s kinetic energy
(ii) the object’s speed
(iii) the object’s velocity
(iv) both (i) and (ii) but not (iii)
(v) both (i) and (iii) but not (ii)
(vi) both (ii) and (iii) but not (i)
(vii) all of (i), (ii), and (iii)
(viii) none of (i), (ii), or (iii)

(CONTINUED ON NEXT PAGE)
3. (continued)

(d) Blocks I and II, each with a mass of 1.0 kg, are hung from the ceiling of an elevator by ropes 1 and 2. The ropes have negligible mass. What is the force exerted by rope 1 on block I when the elevator is traveling upward at a constant speed of 2.0 m/s? (Hint: Use $g = 10 \text{ m/s}^2$.)

(i) 2 N  
(ii) 10 N  
(iii) 12 N  
(iv) 20 N  
(v) 22 N

(e) A 2.0-kg block slides to the left at 3.0 m/s on a frictionless horizontal surface. It collides with a spring of force constant 8.0 N/m. The spring brings the block to a momentary halt. How far does the block compress the spring?

(i) 0.75 m  
(ii) 1.5 m  
(iii) 6.0 m  
(iv) 12 m

END OF THE EXAM
FORMULA SHEET — Page 1 of 2

You may tear this sheet out of the exam — you do not have to hand it in with the rest of your exam.

\[
R_i = A_i + B_i, \quad R_j = A_j + B_j, \quad R_k = A_k + B_k
\]
(components of \( \vec{R} = \vec{A} + \vec{B} \))

\[
\vec{A} = A_i \hat{i} + A_j \hat{j} + A_k \hat{k}, \quad \vec{B} = B_i \hat{i} + B_j \hat{j} + B_k \hat{k}
\]

\[
\vec{A} \cdot \vec{B} = AB \cos \phi = \left| \vec{A} \right| \left| \vec{B} \right| \cos \phi
\]
(definition of the scalar (dot) product)

\[
\vec{A} \cdot \vec{B} = A_i B_i + A_j B_j + A_k B_k
\]
(scalar (dot) product in terms of components)

\[
C = AB \sin \phi \quad (\text{magnitude of } \vec{C} = \vec{A} \times \vec{B})
\]

\[
C_i = A_i B_j - A_j B_i \quad (\text{components of } \vec{C} = \vec{A} \times \vec{B})
\]

\[
v_{av-x} = \frac{x_2 - x_1}{t_2 - t_1} = \Delta x
\]
(average velocity, straight-line motion)

\[
v_s = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}
\]
(instantaneous velocity, straight-line motion)

\[
a_{av-x} = \frac{v_{av-x}}{t_2 - t_1} = \frac{\Delta v}{\Delta t}
\]
(average acceleration, straight-line motion)

\[
a_i = \lim_{\Delta t \to 0} \frac{\Delta v_i}{\Delta t} = \frac{dv_i}{dt}
\]
(instantaneous acceleration, straight-line motion)

\[
v_i = v_0 + a_i t \quad (\text{constant acceleration only})
\]

\[
x = x_0 + v_0 t + \frac{1}{2} a_i t^2 \quad (\text{constant acceleration only})
\]

\[
v^2 = v_0^2 + 2a_i (x - x_0) \quad (\text{constant acceleration only})
\]

\[
x - x_0 = \left( \frac{v_0 + v_i}{2} \right) t \quad (\text{constant acceleration only})
\]

\[
\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}
\]

\[
\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t}
\]
(average velocity vector)

\[
\hat{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}
\]
(instantaneous velocity vector)

\[
v_s = \frac{dx}{dt} \quad \hat{v}_y = \frac{dy}{dt} \quad \hat{v}_z = \frac{dz}{dt}
\]
(components of instantaneous velocity)

\[
\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}
\]
(average acceleration vector)

\[
\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}
\]
(instantaneous acceleration vector)

\[
a_x = \frac{dv_x}{dt} \quad a_y = \frac{dv_y}{dt} \quad a_z = \frac{dv_z}{dt}
\]
(components of instantaneous acceleration)

\[
x = (v_0 \cos \alpha_0) t \quad (\text{projectile motion})
\]

\[
y = (v_0 \sin \alpha_0) t - \frac{1}{2} gt^2 \quad (\text{projectile motion})
\]

\[
v_x = v_0 \cos \alpha_0 \quad (\text{projectile motion})
\]

\[
v_y = v_0 \sin \alpha_0 - gt \quad (\text{projectile motion})
\]

\[
a_{rad} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2} \quad (\text{uniform circular motion})
\]

\[
v_{F/A-x} = v_{F/B-x} + v_{B/A-x} \quad (\text{relative velocity along a line})
\]

\[
\vec{v}_{F/A} = \vec{v}_{F/B} + \vec{v}_{B/A} \quad (\text{relative velocity in space})
\]

\[
\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum \vec{F} \quad (\text{net force})
\]

\[
\sum \vec{F} = 0 \quad \text{or } \Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0
\]
(Newton’s 1st law, body in equilibrium)

\[
\sum \vec{F} = ma \quad \text{or } \Sigma F_x = ma_x, \Sigma F_y = ma_y, \Sigma F_z = ma_z
\]
(Newton’s 2nd law)

\[
w = mg \quad (\text{magnitude of the weight of a body of mass } m)
\]

\[
\vec{F}_{AonB} = -\vec{F}_{BonA} \quad (\text{Newton’s 3rd law})
\]

\[
f_k = \mu_k n \quad (\text{magnitude of kinetic friction force})
\]

\[
f_s \leq \mu_s n \quad (\text{magnitude of static friction force})
\]

\[
W = F_s \cos \phi = \vec{F} \cdot \hat{s} \quad (\text{constant force, straight-line displacement})
\]

\[
K = \frac{1}{2} mv^2 \quad (\text{definition of kinetic energy})
\]

\[
W_{tot} = K_2 - K_1 = \Delta K \quad (\text{work-energy theorem})
\]

\[
W = \int_{t_1}^{t_2} F_x \, dx \quad (\text{straight-line displacement})
\]

\[
W = \int_{t_1}^{t_2} F \cos \phi \, dl = \int_{t_1}^{t_2} F_x \, dl = \int_{t_1}^{t_2} \vec{F} \cdot d\vec{l}
\]
(curved path)

\[
P_{av} = \frac{\Delta W}{\Delta t} \quad (\text{average power})
\]

\[
P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} \quad (\text{instantaneous power})
\]

\[
P = \vec{F} \cdot \hat{v} \quad (\text{instantaneous rate at which force } \vec{F} \text{ does work on a particle})
\]

(continued on next page)
You may tear this sheet out of the exam — you do not have to hand it in with the rest of your exam.

\[ W_{\text{grav}} = F_s = w (y_1 - y_2) = mgy_1 - mgy_2 \]
\[ W_{\text{grav}} = U_{\text{grav},1} - U_{\text{grav},2} = -(U_{\text{grav},2} - U_{\text{grav},1}) = -\Delta U_{\text{grav}} \]
(work done by the gravitational force)
\[ U_{\text{grav}} = mgy \quad \text{(gravitational potential energy)} \]
\[ W_{\text{el}} = \frac{1}{2} k x_1^2 - \frac{1}{2} k x_2^2 = U_{\text{el},1} - U_{\text{el},2} = -\Delta U_{\text{el}} \]
(work done by the elastic force)
\[ U_{\text{el}} = \frac{1}{2} k x^2 \quad \text{(elastic potential energy)} \]
\[ K_1 + U_1 = K_2 + U_2 \]
(if only gravity and elastic forces do work)
\[ K_1 + U_1 + W_{\text{other}} = K_2 + U_2 \]
(if other forces also do work)

\[ \sin \theta = \frac{B}{C} \]
\[ \cos \theta = \frac{A}{C} \]
\[ \tan \theta = \frac{B}{A} \]

The graph of \( y = \sin \theta \)

The graph of \( y = \cos \theta \)

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FoxTrot by Bill Amend

"Physics Always Lose Snowball Rights."