An Interesting Projectile Problem

Want a projectile to reach location \((x, y)\) relative to launch point origin. Initial angle \(\theta_0\) is given. Want to find magnitude of initial velocity \(v_0\).

\[
v_0 = v_0 \cos \theta_0
\]

\[
v_0 = v_0 \sin \theta_0
\]

\[
x = v_0 \cos \theta_0 \ t \rightarrow t = \frac{x}{v_0 \cos \theta_0}
\]

\[
y = v_0 \sin \theta_0 \ t - \frac{g}{2} t^2
\]

\[
y = v_0 \sin \theta_0 \left(\frac{x}{v_0 \cos \theta_0}\right) - \frac{g}{2} \left(\frac{x}{v_0 \cos \theta_0}\right)^2
\]

\[
y = x \tan \theta_0 - \frac{g x^2}{2 v_0^2 \cos^2 \theta_0}
\]

\[
\frac{g}{2} \frac{x^2}{v_0^2 \cos^2 \theta_0} = x \tan \theta_0 - y
\]

\[
\frac{2 v_0^2 \cos^2 \theta_0}{g x^2} = \frac{1}{x \tan \theta_0 - y}
\]

\[
v_0^2 = \frac{g x^2}{2 \cos^2 \theta_0 (x \tan \theta_0 - y)}
\]

\[
v_0 = \frac{x}{\cos \theta_0} \sqrt{\frac{g}{2 (x \tan \theta_0 - y)}}
\]

Note: need \(x \tan \theta_0 - y > 0\)
A similar problem can be solved in which the magnitude of initial velocity \( v_0 \) is given but the launch angle \( \theta_0 \) is desired.

The problem solution is the same up to the Eqn. 1 of the previous. Then \( \frac{1}{\cos^2 \theta_0} \) is replaced:

\[
\frac{1}{\cos^2 \theta_0} = \sec^2 \theta_0 = \tan^2 \theta_0 + 1
\]

So obtain:

\[
y = x \tan \theta_0 - \frac{g x^2}{2 v_0^2} \tan \theta_0 - \frac{g x^2}{2 v_0^2} \tan^2 \theta_0 - x \tan \theta_0 + \left( y + \frac{g x^2}{2 v_0^2} \right) = 0
\]

This is a quadratic in \( \tan \theta_0 \),

let \( \eta = \tan \theta_0 \)

\[
\left( \frac{g x^2}{2 v_0^2} \right) \eta^2 - x \eta + \left( y + \frac{g x^2}{2 v_0^2} \right) = 0
\]

Solve for \( \eta \), then \( \theta_0 = \tan^{-1}(\eta) \)

Note, it is common to obtain two positive values of \( \eta \) and thus two angles \( \theta_0 \) in the range \( 0 < \theta_0 < 90^\circ \).