Jumping on a Merry-go-Round

A girl $m_g = 30 \text{ kg}$ is running at $2 \text{ m/s}$ and jumps on a turning merry-go-round (MGR). The MGR (modeled as a solid disc) has $M = 200 \text{ kg}$ and $R = 2 \text{ m}$. It turns at $1 \text{ rad/s}$ (clockwise), thus $\omega_{MGR} = -1 \text{ rad/s}$. The MGR turns toward the girl. Her velocity path makes an angle of $30^\circ$ with respect to a tangent to the MGR. See Figure. She lands at a point $r = 1.5 \text{ m}$ from the MGR's axis of rotation. Ignore any friction at the axis of rotation.

What is the angular velocity of the MGR with girl after she jumps on?

This is an angular collision. Since $T = 0$, the angular momentum is conserved. The angular momentum is calculated with respect to the axis of rotation.

\[ \Delta L = 0 \rightarrow L_i = L_f \]

The initial angular momentum is the sum of the MGR's $L_{MGR} = I_{MGR} \omega_i$ and the $L_g$ of the girl.

\[ I_{MGR} = \frac{1}{2} MR^2 = \frac{1}{2} (200 \text{ kg})(2 \text{ m})^2 = 400 \text{ kgm}^2 \]

\[ L_{MGR} = 400 \text{ kgm}^2 (-1 \text{ rad/s}) = -400 \text{ kgm}^2/\text{s} \]

(\(-2\) direction)

The girl has linear momentum of magnitude

\[ p_g = m_g v_g = 30 \text{ kg} \times 2 \text{ m/s} = 60 \text{ kgm/s}. \]
Her initial angular momentum with respect to the axis of rotation is
\[ \vec{L}_g = \vec{r} \times \vec{p}_g \], \quad L_g = r \cdot p \cdot \sin \Theta

\Theta \) is the angle between the \( \vec{r} \) direction and her \( \vec{p}_g \) direction. The direction of \( L_g \) is given by the right-hand sign rule. From the figure, we see \( \Theta = 120^\circ \).

Thus \[ L_g = 1.5 m (60 \text{ kg} \cdot \text{m/s}) \sin 120^\circ = 77.9 \text{ kg} \cdot \text{m}^2/\text{s} \]
The direction is counterclockwise, i.e., positive. (\( +z \) direction)

Let's use \[ \vec{L}_g = \vec{r} \times \vec{p}_g \], \[ \vec{r} = 1.5 \hat{i} + 0 \hat{j} + 0 \hat{k} \]
\[ \vec{p}_g = -p \sin 30 \hat{i} + p \cos 30 \hat{j} + 0 \hat{k} \]
\[ = -60 \sin 30 \hat{i} + 60 \cos 30 \hat{j} + 0 \hat{k} \]
\[ \vec{r} \times \vec{p}_g = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.5 & 0 & 0 \\ -30 & 52 & 0 \end{vmatrix} = (0 \hat{i} + 0 \hat{j} + 1.5 \times 52 \hat{k} \]
\[ = 0 \hat{i} + 0 \hat{j} + 77.9 \hat{k} \]
in agreement.

\( \vec{L}_f = -400 \text{ kg} \cdot \text{m}^2/\text{s} + 77.9 \text{ kg} \cdot \text{m}^2/\text{s} = -322.1 \text{ kg} \cdot \text{m}^2/\text{s} \)

\( I_f = I_i = -322.1 \text{ kg} \cdot \text{m}^2/\text{s} = I_{\text{total}} \cdot \omega_f \)
\( \omega_f = \frac{L_f}{I_{\text{total}}} \)

\( I_{\text{total}} = I_{\text{MGR}} + I_g \)
\( I_{\text{MGR}} = 400 \text{ kg} \cdot \text{m}^2 \) (no change)

\( I_g \) is the moment of inertia of a point mass
\[ I_g = mg(r)^2 = 30 \text{ kg} \times (1.5 \text{ m})^2 = 67.5 \text{ kg} \cdot \text{m}^2 \]

\( I_{\text{total}} = 400 \text{ kg} \cdot \text{m}^2 + 67.5 \text{ kg} \cdot \text{m}^2 = 467.5 \text{ kg} \cdot \text{m}^2 \)

Thus \[ \omega_f = \frac{L_f}{I_{\text{total}}} = -\frac{322.1 \text{ kg} \cdot \text{m}^2/\text{s}}{467.5 \text{ kg} \cdot \text{m}^2} = -0.689 \text{ rad/s} \]