Linear and Rotational Motion

Flywheel \( M = 10.0 \text{kg} \)
Uniform disk, No slip of cable

\( R = 0.200 \text{m} \)

\( M_1 = 10.0 \text{ kg}, \ m_2 = 2.00 \text{ kg} \)

\( I = \frac{1}{2} MR^2 \)

Starts at rest. Does system remain at rest?

Forces on \( m_2 \):

\[ T_2 = m_2g = 2 \text{ kg} \times 9.8 \text{ m/s}^2 = 19.6 \text{ N} \]

If no motion, \( T_1 = T_2 = 19.8 \text{ N} \)

**F.B.D. of \( m_1 \):**

\[ F_s \]

\[ n - W \cos \theta = 0 \]

\[ n = W \cos \theta = m_1g \cos \theta = 10.0 \text{ kg} \times 9.8 \text{ m/s}^2 \cos 30^\circ \]

\[ n = 84.87 \text{ N} \]

\[ F_{s_{\text{max}}} = \mu_s n = 0.30 \times 84.87 \text{ N} = 25.46 \text{ N} \]

\[ \sum F_y = 0 \]

\[ m_1g \sin \theta - T_1 - F_s = 0 \]

\[ F_s = m_1g \sin \theta - T_1 \]

\[ = 10.0 \text{ kg} \times 9.8 \text{ m/s}^2 \times \sin 30^\circ - 19.8 \text{ N} \]

\[ = 29.2 \text{ N} \]

\( F_s \) needs to be 29.2N to prevent motion, but \( F_{s_{\text{max}}} = 25.5 \text{ N} \). Thus system moves.

Thus friction force is

\[ F_k = \mu_k n = \mu_k m_1g \cos \theta \]

\[ = 0.25 \times 84.87 \text{ N} \]

\[ = 21.22 \text{ N} \]
There will be acceleration of $m_1$ down the ramp and $m_2$ up and flywheel will have an angular acceleration $\alpha$. This requires a torque on the flywheel, so $T_1 = T_2$

For $m_1$ with a down the ramp:
$$m_1 g \sin \theta - T_1 - F_k = m_1 a \quad (1)$$

For the flywheel $\Sigma T = I \alpha$
$$T_1 R - T_2 R = I \alpha$$
No slip of cable on the flywheel results in $a = R \alpha$, thus $\alpha = \frac{a}{R}$
So $T_1 R - T_2 R = \frac{1}{2} MR^2 \frac{a}{R}$
Becomes $T_1 - T_2 = \frac{1}{2} MA \quad (2)$

For $m_2$ with a acceleration up
$$T_2 - m_2 g = M_2 a \quad (3)$$
Adding equations (1), (2) and (3) eliminates $T_1$ and $T_2$, giving:
$$m_1 g \sin \theta - F_k - m_2 g = (m_1 + m_2 + \frac{M}{2}) a$$
Since $F_k = \mu_k m_1 g \cos \theta$
$$a = \frac{g \left( m_1 \sin \theta - m_2 - \mu_k m_1 \cos \theta \right)}{m_1 + m_2 + \frac{M}{2}}$$
$$a = \frac{9.8 m/s^2 \left( 10 kg \sin 30^\circ - 2 kg - 25 \times 70 kg \cos 30^\circ \right)}{10 kg + 2 kg + \frac{10 kg}{2}}$$
\[ a = \frac{8.183 \text{ N}}{17 \text{ kg}} = 0.481 \text{ m/s}^2 \]

When \( t = 4.00 \text{ s} \), \( v = v_0 + at = 0.481 \times 4 \)

\[ \frac{a}{v_0} t + \frac{v_0 t^2}{2} = 1.925 \text{ m/s} \]

\[ x - x_0 = v_0 t + \frac{a t^2}{2} = 0.481 \times (4.0)^2 = 3.848 \text{ m} \]

down the ramp for \( m_1 \) and up \( 3.848 \text{ m} \) for \( m_2 \)

Now let's analyze using Work/Energy with the understanding that it does move. We will start with a given that the system has moved from rest the above calculated distance and find the velocity.

\[ W_{nc} = \Delta E \quad E = K_f + K_f + U_g + U_{sp} \]

For \( m_1 \), let \( y_{1i} = 0 \), \( y_{1f} = -3.848 \sin 30 \)

\[ = -1.924 \text{ m} \]

For \( m_2 \), let \( y_{2i} = 0 \), \( y_{2f} = 3.848 \text{ m} \)

\[ v_{z} = 0 \text{ and } \omega_{z} = 0 \]

Thus \( E_{z} = 0 \)

For \( E_f \):

\[ K_f = \frac{1}{2} m_1 v_{f}^2 + \frac{1}{2} m_2 v_{f}^2 + \frac{1}{2} I \omega_f^2 \]

No slip of cable on flywheel gives

\[ v = \omega R \], \text{ so } \omega = \frac{v}{R} \text{, } \omega_f = \frac{v_f}{R} \]

Thus \( \frac{1}{2} I \omega_f^2 = \frac{1}{2} \left( \frac{1}{2} MR^2 \right) \left( \frac{v_f}{R} \right)^2 = \frac{1}{4} M v_f^2 \)

And \( E_f \) is

\[ E_f = K_{Total} + U_{g_{m_1}} + U_{g_{m_2}} \]

\[ = \frac{1}{2} m_1 v_{f}^2 + \frac{1}{2} m_2 v_{f}^2 + \frac{1}{4} M v_f^2 + m_1 g y_{1f} + m_2 g y_{2f} \]
\[ E_f = U_f \left( \frac{m_1}{2} + \frac{m_2}{2} + \frac{M}{4} \right) + m_1g y_{if} + m_2g y_{zf} \]

\[ W_{nc} = -F_R \cdot \text{distance} = -\mu_k m_1g \cos 30 \times 3.848 \text{m} \]
\[ = -0.25 \times 10 \text{kg} \times 9.8 \text{m/s}^2 \times \cos 30 \times 3.848 \text{m} \]
\[ = -81.64 \text{J} \]

\[ W_{nc} = E_f - E_i, \text{ Remember we have } E_i = 0 \]
So \[ -81.64 \text{J} = U_f \left( \frac{m_1}{2} + \frac{m_2}{2} + \frac{M}{4} \right) + m_1g y_{if} + m_2g y_{zf} \]

\[ U_f = \sqrt{\frac{-81.64 - m_1g y_{if} - m_2g y_{zf}}{m_1 + m_2 + M/4}} \]

\[ = \sqrt{\frac{-81.64 - 10 \times 9.8 \times (-1.924 \text{m}) - 2 \times 9.8 \times 3.848 \text{m}}{10 \text{kg}/z + 2 \text{kg}/z + 10 \text{kg}/y}} \]
\[ = \sqrt{\frac{31.49 \text{J}}{8.51 \text{kg}}} = 1.925 \text{ m/s} \]

Thus after moving 3.848 m from rest system has velocity 1.925 m/s.

Since all forces are constant, there must be constant acceleration, thus kinematics may be used and

\[ v^2 = v_0^2 + 2a(x-x_0) \rightarrow a = \frac{v^2}{2(x-x_0)} \]

\[ a = \left( \frac{1.925 \text{m/s}}{2 \times 3.848 \text{m}} \right)^2 = 0.481 \text{m/s}^2 \]

and \[ v = v_0 + at \rightarrow t = \frac{v}{a} \]

\[ t = \frac{1.925 \text{m/s}}{0.481 \text{m/s}^2} = 4.00 \text{ s} \]

All in agreement.