A 4-kg block hangs by a light string that passes over a smooth peg and is connected to a 6-kg block that rests on a rough table (Figure 6-25). The coefficient of kinetic friction is $\mu_k = 0.2$. The 6-kg block is pushed against a spring, which has a force constant of 180 N/m, compressing it 30 cm. It is then released. Find the speed of the blocks when the 4-kg block has fallen a distance of 40 cm.

We consider the system consisting of the spring and the two masses which are connected by a light string which does not stretch. The spring is not connected to $m_1$. There is friction only between $m_1$ and the table.

At the initial condition the masses are at rest and the spring is compressed 30 cm, $\Delta x_i = -0.30\text{ m}$. Mass $m_1$ does not change height and $m_2$ is taken as $y_i = 0$.

The initial energy is

\[ E_i = K_i + U_{gi} + U_{spi} \]
\[ = 0 + 0 + \frac{1}{2} k \Delta x_i^2 \]
\[ = \frac{1}{2} (180 \text{ N/m}) (-0.30 \text{ m})^2 \]
\[ = 8.1 \text{ J} \]

At the final condition the two masses are moving with the same unknown speed $v_f$. The spring has its natural length so $\Delta x_f = 0$. Mass $m_2$ has moved.
downward and has \( y_f = -40\text{cm} = -0.40\text{m} \).

Note, \( y \) for gravitational potential energy is always positive in the direction opposite of \( g \).

The final energy:
\[
E_f = K_f + U_{gf} + U_{spf}
= \frac{1}{2} (m_1 + m_2) \frac{v_f^2}{2} + m_2 g y_f + 0
= \frac{1}{2} (6.0\text{kg} + 4.0\text{kg}) \frac{v_f^2}{2} + 4.0\text{kg} (9.8\text{m/s}^2) (-0.40\text{m})
= 5\text{kg} \frac{v_f^2}{2} - 15.7 \text{J}
\]

The kinetic friction acting on \( m_i \), has magnitude \( F_K = \mu_k n \), but \( n = m_i g \), so
\( F_K = \mu_k m_i g \). This force acts toward the left while \( m_i \) is moving distance
\( x_i = 0.40\text{m} \) toward the right

\[
\text{Work}_{NC} = F_K x_i \cos 180^\circ = -\mu_k m_i g x_i,
\]
\[
\text{Work}_{NC} = -0.20 (6.0\text{kg})(9.8\text{m/s}^2) (0.40\text{m})
= -4.7 \text{J}
\]

Since \( \text{Work}_{NC} = \Delta E = E_f - E_i \)
we have:
\[
E_f = \text{Work}_{NC} + E_i
\]
\[
5\text{kg} \frac{v_f^2}{2} - 15.7 \text{J} = -4.7 \text{J} + 8.1 \text{J}
\]
\[
5\text{kg} \frac{v_f^2}{2} = 19.1 \text{J}
\]
\[
\frac{v_f^2}{5\text{kg}} = 3.82 \text{m}^2/\text{s}^2
\]
\[
v_f = \sqrt{3.82 \text{m}^2/\text{s}^2} = 1.95 \text{m/s}
\]