Spring-Mass System

Horizontal

\[ x \text{ is displacement from equilibrium} \]

If friction = 0

then \( W_{other} = 0 \) and \( \Delta E = 0 \)

\[ K + U_{el} = \text{Constant} \]

\[ \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \text{Constant} \]

When \( x = 0 \), \( v = \pm v_{max} \) and Constant = \( \frac{1}{2} m (v_{max})^2 \)

When \( V = 0 \), \( x = \pm x_{max} \) and Constant = \( \frac{1}{2} k (x_{max})^2 \)

The motion is Simple Harmonic Motion

If initial condition at \( t=0 \) is \( x=A, v=0 \)

then \( x = A \cos \omega t \), \( \omega = \sqrt{\frac{k}{m}} \)

\( v = -\omega A \sin \omega t \)

\( a = -\omega^2 A \cos \omega t \)

So \( x_{max} = A \), \( v_{max} = \omega A \), \( a_{max} = \omega^2 A \)
When \( W_0 = 0 \), \( \Delta E = 0 \)

So \( E = K + U_{el} + U_g = \text{Constant} \)

\( K = \frac{1}{2} m V^2 \), here \( V > 0 \) down, direction of \( y > 0 \)

\( U_{el} = \frac{1}{2} k (y_e + y)^2 \), Spring stretched amount \( y_e + y \)

\( U_g = -mg \left( \frac{y_e}{2} + y \right) \), Negative since \( \frac{y_e}{2} + y \)

is sensed down, the \( \frac{y_e}{2} \) is convenient for what follows but remember that the origin is arbitrary for \( U_g \).

Thus:

\[
E = \frac{1}{2} m V^2 + \frac{1}{2} k (y_e + y)^2 - mg \left( \frac{y_e}{2} + y \right)
\]

\[
= \frac{1}{2} m V^2 + \frac{1}{2} k \left( y_e^2 + 2 y_e y + y^2 \right) - k y_e \left( \frac{y_e}{2} + y \right)
\]

\[
= \frac{1}{2} m V^2 + \frac{1}{2} k y_e^2 + k y_e y + \frac{1}{2} k y^2 - k y_e \frac{y_e}{2} - k y_e y
\]

\[
= \frac{1}{2} m V^2 + \frac{1}{2} k y^2
\]

What has happened is fantastic!

We can ignore gravity and thus \( U_g \) if we use the displacement, \( y \), from the equilibrium location in the spring energy \( U_{el} \).

Vertical Simple Harmonic Motion is exactly like horizontal simple harmonic motion when \( y \) is displacement from vertical equilibrium.