Problem 5.81  Crate on Ramp With Hanging Washer

A steel washer is suspended inside an empty shipping crate from a light string attached to the top of the crate. The crate slides down a long ramp that is inclined at an angle of 35° above the horizontal. The crate has mass 156 kg. You are sitting inside the crate (with a flashlight); your mass is 56 kg. As the crate is sliding down the ramp, you find the washer is at rest with respect to the crate when the string makes an angle of 74° with the top of the crate.

Part A

What is the coefficient of kinetic friction between the ramp and the crate?

ANSWER:

✓ Correct

\[ \mu_k = 0.287 \]

Since the washer is at rest relative to the crate, they must have the same acceleration. The crate can only have acceleration along the ramp direction. Thus analyze the hanging washer for its acceleration in the ramp direction.

For washer \( a_y = 0 \)
\[ a_x = \text{desired value} \]
\[ \Sigma F_y = 0 \]
\[ T \cos 16° - W \cos 35° = 0 \]
\[ T = W \frac{\cos 35°}{\cos 16°} \]
\[ \Sigma F_x = \max \]
\[ W \sin 35° - T \sin 16° = \max \]
\[ W \sin 35° - W \frac{\cos 35°}{\cos 16°} \sin 16° = \max \]
\[ W \sin 35° - W \cos 35° \tan 16° = \max \]
but \( W = mg \) so
\[ mg \sin 35° - mg \frac{\cos 35°}{\cos 16°} \tan 16° = \max \]
\[ a_y = g (\sin 35° - \cos 35° \tan 16°) \]
\[ a_x = g (0.3387) = 9.8 \frac{m}{s^2} (0.3387) \]
\[ a_x = 3.319 \frac{m}{s^2} \]
Now analyze the crate with you inside.
Total mass \( m = 152 \, \text{kg} + 56 \, \text{kg} = 212 \, \text{kg} \)
accelerating at \( 3.319 \, \text{m/s}^2 \) down the ramp.

\[ \sum F_y = 0 \]
\[ n = \dot{W} \cos 35^\circ = 0 \]
\[ n = \dot{W} \cos 35^\circ \]
\[ n = 212 \, \text{kg} \times 9.8 \, \text{m/s}^2 \times \cos 35^\circ \]
\[ n = 1702 \, \text{N} \]

\[ F_K = \mu_k \, n \]

\[ \sum F_x = m a_x \]
\[ \dot{W} \sin 35^\circ - F_K = m a_x \]
\[ m g \sin 35^\circ - \mu_k \, n = m a_x \]
\[ \mu_k \, n = m (g \sin 35^\circ - a_x) \]
\[ \mu_k = \frac{m (g \sin 35^\circ - a_x)}{n} \]
\[ \mu_k = \frac{212 \, \text{kg} (9.8 \, \text{m/s}^2 \times \sin 35^\circ - 3.319 \, \text{m/s}^2)}{1702 \, \text{N}} \]
\[ \mu_k = 0.287 \]