11.94 Knocking Over a Post. One end of a post weighing 400 N and with height \( h \) rests on a rough horizontal surface with \( \mu_s = 0.30 \). The upper end is held by a rope fastened to the surface and making an angle of 36.9° with the post (Fig. 11.54). A horizontal force \( F \) is exerted on the post as shown. a) If the force \( F \) is applied at the midpoint of the post, what is the largest value it can have without causing the post to slip? b) How large can the force be without causing the post to slip if its point of application is \( \frac{h}{3} \) of the way from the ground to the top of the post? c) Show that if the point of application of the force is too high, the post cannot be made to slip, no matter how great the force. Find the critical height for the point of application.

**Figure 11.54 Challenge Problem 11.94.**

\[
\sin 36.9^\circ = 0.60 \\
\cos 36.9^\circ = 0.80
\]

\[
\text{In Equilibrium:} \\
\Sigma T_x = 0 \\
F \frac{h}{2} - F_s h = 0 \\
F = 2F_s \tag{1}
\]

\[
\Sigma F_y = 0 \\
n - W - \frac{4}{3}T = 0 \\
n = W + \frac{4}{3}T \tag{2}
\]

\[
\Sigma F_x = 0 \\
F - F_s - \frac{3}{5}T = 0 \\
F = \frac{3}{5}T + F_s \tag{3}
\]

Maximum \( F_S \), \( F_s = F_{S_{\text{max}}} = \mu_s n \tag{4} \)

**Known:** \( \mu_s = 0.30 \), \( W = 400 \text{N} \) **Unknowns:** \( F, F_s, n, T \)

Using (2) and (3) eliminate \( T \), obtain

\[
n = W + \frac{4}{3}F - \frac{4}{3}F_s \tag{5}
\]

Using (4) and (5) eliminate \( n \)

\[
F_s = \mu_s \left( W + \frac{4}{3}F - \frac{4}{3}F_s \right) \\
\text{and isolate } F_s
\]

\[
F_s = \frac{\mu_s}{1 + \frac{4}{3}\mu_s} \left( W + \frac{4}{3}F \right) \tag{6}
\]
Using (1) and (2) eliminate $F_s$

$$F = \frac{2\mu_s}{1 + \frac{4}{3}\mu_s} \left( W + \frac{4}{3} F \right)$$

and isolate $F$

$$F = \left( \frac{2\mu_s}{1 - \frac{4}{3}\mu_s} \right) W = \frac{2(\cdot3)}{1 - \frac{4}{3}(\cdot3)} W = W$$

Part (a) $F = W = 400$ N

Part (c) Let $F$ be applied a distance $d = xH$, $0 < x \leq 1$ below the top, i.e. below point $O$ for torque calculation.

Eqn (1) becomes $F_x h - F_s h = 0$

$$F = \frac{F_s}{x} \quad (2)$$

Equations (2), (3), (4) do not change. Thus equations (5) and (6) do not change.

Combine (6) and (7) $F = \frac{\mu_s}{x} \left( W + \frac{4}{3} F \right)$

and isolate $F$, obtain

$$F = \frac{\mu_s / x}{1 + \frac{4}{3}\mu_s - \frac{4}{3}\frac{\mu_s}{x}} W$$

The denominator, i.e. $1 + \frac{4}{3}\mu_s - \frac{4}{3}\frac{\mu_s}{x}$

becomes zero for

$$x = \frac{4\mu_s}{3 + 4\mu_s}$$

For $\mu_s = 0.30$, $x = 0.286$

This allows $F$ to be arbitrarily large and $F_s$ not needing its maximum value $F_{s,\text{max}}$