Incline Plane and Similar Problems

When one or more objects move on a non-horizontal plane, the following points should be noted:

- Gravity, and thus weight forces, always act down. (Only a component of weight acts against plane.)
- Normal forces always act perpendicular to the plane on which an object moves.
- Friction forces always act parallel to the plane on which an object moves.
- Separate x-y coordinate systems should be used for each object so that the object moves in only one coordinate direction.
- Motion in the separate x-y coordinate systems are geometrically related to each other.
- Strings passing over ideal frictionless pulleys have the same tension on both sides of the pulley.

Example:

Pulley is ideal (no mass or friction)
Incline has no friction
Blocks connected by string

$m_2 = 7.0 \text{ kg}, \theta = 40^\circ$

For what value of $m_1$, is the system in equilibrium?

Equilibrium requires

$\sum F = 0$

$\sum F_x = 0$, $\sum F_y = 0$, $\sum F_z = 0$
FBD for $m_1$:

\[ \begin{array}{c}
\uparrow T_1 \\
\downarrow m_1 \\
\downarrow w_1 = m_1 g \\
\end{array} \]

\[ T_1 = m_1 g \]

\[ m_1 = \frac{T_1}{g} \]

FBD for $m_2$:

\[ \begin{array}{c}
\uparrow T_2 \\
\downarrow n_2 \\
\downarrow T_1 \\
\downarrow w_2 = m_2 g \\
\end{array} \]

\[ T_2 - w_2 \sin \theta = 0 \]

But $T_1 = T_2$ (Ideal pulley)

Thus

\[ m_1 = \frac{T_2}{g} = \frac{m_2 g \sin \theta}{g} = m_2 \sin \theta \]

\[ m_1 = 7.0 \text{ kg} \times \sin 40^\circ = 4.5 \text{ kg} \]

Continue:

If $m_1 = 6.0 \text{ kg}$, what would happen?

FBD for $m_1$:

\[ \begin{array}{c}
\uparrow T_1 \\
\downarrow m_1 \\
\downarrow w_1 = m_1 g \\
\end{array} \]

\[ \Sigma F_y = m_1 a_y \]

\[ w_1 - T_1 = m_1 a_y \] (Notice $y$ chosen down)

FBD for $m_2$:

\[ \begin{array}{c}
\uparrow T_2 \\
\downarrow n_2 \\
\downarrow T_1 \\
\downarrow w_2 = m_2 g \\
\end{array} \]

\[ \Sigma F_x = m_2 a_x \]

\[ T_2 - w_2 \sin \theta = m_2 a_x \]
With the choice of coordinate directions
\[ a_{2x} = a_{1y} = a \]
because the string constrains the motion. Also \( T_1 = T_2 \) because pulley is ideal.
Equations become:
\[ W_1 - T = m_1 a \]
\[ T - W_2 \sin \theta = m_2 a \]
Add \( W_1 - W_2 \sin \theta = (m_1 + m_2) a \)
\[ a = \frac{W_1 - W_2 \sin \theta}{m_1 + m_2} = \frac{g(m_1 - m_2 \sin \theta)}{m_1 + m_2} \]
\[ a = \frac{9.8 \text{ m/s}^2 (6.0 \text{ kg} - 7.0 \text{ kg} \sin 40^\circ)}{6.0 \text{ kg} + 7.0 \text{ kg}} = 1.1 \text{ m/s}^2 \]

\( m_1 \) accelerates down and \( m_2 \) accelerates up the incline.
Notice we could have worked with a single \( T \) and a single \( a \) from the beginning as long as we clearly understood why. We would choose directions:

![Diagram](image)

Example: \( \mu_s, \mu_k \)

At what minimum acceleration \( a \) will the mass \( M \) not fall?
If the mass $M$ is accelerating then $m$ is accelerating.

If $m$ is accelerating there must be a force on $m$ to cause the acceleration.

That force is a normal force of $M$ acting on $m$.

If there is a normal force there is a static friction force which can act to offset the weight of $m$.

Thus $m$ can be in equilibrium for vertical motion.

**FBD of $m$:**

![FBD Diagram]

- **$w = mg$**

**X-Direction**

**NL2:** $\Sigma F_x = ma$

$n = ma$

**Y-Direction**

**NL1:** $\Sigma F_y = 0$

$F_s - w = 0 \Rightarrow F_s = w$

But $F_s \leq \mu_s n$

So $w \leq \mu_s ma \Rightarrow a \geq \frac{w}{\mu_s m} = \frac{mg}{\mu_s m} = \frac{g}{\mu_s}$

$a \geq \frac{g}{\mu_s}$