Combining Vectors

1. Orient all vectors in an x-y coordinate system.

2. Characterize each vector $\vec{V}$ by its magnitude $V = |\vec{V}|$ and its direction angle $\theta$. $\theta$ is the counterclockwise angle from the positive x-axis to the arrow end of the vector. $0^\circ \leq \theta \leq 360^\circ$

3. Find the x and y components of each vector
   
   $V_x = V \cos \theta$
   $V_y = V \sin \theta$

   (Reverse signs for any vector being subtracted.)

4. The x and y components of the resultant vector are the sums of the components of the vectors being combined.

   $R_x = \sum V_x \quad R_y = \sum V_y$

5. The resultant vector $\vec{R}$ is characterized by its magnitude

   $|\vec{R}| = R = \sqrt{R_x^2 + R_y^2}$

   and its direction angle $\theta$.

   To find $\theta$, plot the $R_x$ and $R_y$ values in the original x-y system and approximately draw the vector $\vec{R}$. Find the related acute angle, $\hat{\theta}$, between the vector $\vec{R}$ and the x-axis.
\[ \hat{\theta} = \tan^{-1} \left| \frac{R_y}{R_x} \right| \]

where only the absolute value of the \( \frac{R_y}{R_x} \) is used. Thus \( 0 \leq \hat{\theta} \leq 90^\circ \).

From your plot, determine angle \( \hat{\theta} \) with orientation required by the problem.

Example cases of \( \hat{\theta} \) and \( \theta \).

**Case a.** \( R_x > 0, \ R_y > 0 \), \( \theta = \hat{\theta} \)

**Case b.** \( R_x > 0, \ R_y < 0 \)
\[ \theta = 360^\circ - \hat{\theta} \]

**Case c.** \( R_x < 0, \ R_y > 0 \)
\[ \theta = 180^\circ - \hat{\theta} \]

**Case d.** \( R_x < 0, \ R_y < 0 \)
\[ \theta = 180^\circ + \hat{\theta} \]

Other special cases:

- \( R_x = 0, \ R_y > 0 \), \( \hat{\theta} \) not calculated but \( \theta = 90^\circ \)
- \( R_x = 0, \ R_y < 0 \), \( \hat{\theta} \) not calculated but \( \theta = 270^\circ \)
- \( R_x > 0, \ R_y = 0 \), \( \hat{\theta} = 0^\circ \quad \theta = 0^\circ \)
- \( R_x < 0, \ R_y = 0 \), \( \hat{\theta} = 0^\circ \quad \theta = 180^\circ \)
- \( R_x = 0, \ R_y = 0 \) \( \theta \) not needed
Example: Given vectors $\vec{A}$ and $\vec{B}$ as shown

\[ \begin{align*}
\vec{A} &= 4.00 \quad \text{(4.00 units along the x-axis)} \\
\vec{B} &= 3.00 \quad \text{(3.00 units along the y-axis)}
\end{align*} \]

Change to:

\[ \begin{align*}
\vec{A} &= 4.00 \quad \text{(4.00 units along the x-axis)} \\
\vec{B} &= 3.00 \quad \text{(3.00 units along the y-axis)}
\end{align*} \]

Calculate components:

\[ \begin{align*}
A_x &= 4.00 \cos 110^\circ = -1.37 \\
A_y &= 4.00 \sin 110^\circ = 3.76 \\
B_x &= 3.00 \cos 280^\circ = 0.52 \\
B_y &= 3.00 \sin 280^\circ = -2.95
\end{align*} \]

Sum to obtain resultant components

\[ \begin{align*}
R_x &= A_x + B_x = -1.37 + 0.52 = -0.85 \\
R_y &= A_y + B_y = 3.76 + (-2.95) = 0.81
\end{align*} \]

Magnitude of resultant $R$

\[ R = \sqrt{(-0.85)^2 + (0.81)^2} = 1.17 \]

\[ \hat{\Theta} = \tan^{-1} \left| \frac{R_y}{R_x} \right| = \tan^{-1} \left| \frac{0.81}{-0.85} \right| = \tan^{-1} (0.953) = 44^\circ \]

Draw vector $\vec{R}$

We see that

\[ \Theta = 180^\circ - \hat{\Theta} = 180^\circ - 44^\circ = 136^\circ \]

(Case c.) with $R_x < 0$, $R_y > 0$)
Practice examples:

For $\vec{R} = \vec{A} + \vec{B}$

$\vec{R} = 2.87$
$\theta = 342^\circ$

For $\vec{R} = \vec{A} - \vec{B}$

$\vec{R} = 8.59$
$\theta = 50.5^\circ$

For $\vec{R} = -\vec{A} + \vec{B}$

$\vec{R} = 6.61$
$\theta = 221^\circ$