1. \[ F = \frac{m}{l} a \]

Method 1: Newton's Laws & Kinematics

\[ F = ma \implies a = \frac{F}{m} \]

\[ v_0 = 0 \]

\[ x - x_0 = d \]

\[ v^2 = 2ad \]

\[ v = \sqrt{\frac{2Fd}{m}} \]

\[ t = \frac{\sqrt{\frac{2Fd}{m}}}{a} \]

Method 2: Work-Energy Theorem & Impulse

\[ F \cdot d = \frac{1}{2}mv^2 \implies v = \sqrt{\frac{2Fd}{m}} \]

\[ F \Delta t = mv \]

\[ F \Delta t = m\sqrt{\frac{2Fd}{m}} \]

\[ \Delta t = \frac{m}{F} \sqrt{\frac{2Fd}{m}} = \sqrt{\frac{2md}{F}} \]

2. From Work-Energy Theorem:

\[ F \cdot d = \Delta K \]

From Impulse-Momentum Theorem:

\[ \Delta p = F \Delta t \]

a) If \( d \) is the same for both masses, both will have the same kinetic energy. The larger mass will require more time, so will have greater momentum.

b) If \( \Delta t \) is the same for both masses, the momenta will be the same. Since the smaller mass will have moved a longer distance during this time, the smaller mass will have greater kinetic energy.
First we need to find the speed of the golf ball after the club hits it.

From projectile motion:

\[ z_0 m = v_{0x} t \]

\[ 0 = v_{0y} t - \frac{1}{2} g t^2 \]

Let's solve for \( t \) to find the time the ball is in the air.

\[ 0 = t \left( v_{0y} - \frac{1}{2} g t \right) \quad \text{we don't want } t = 0, \text{ so} \]

\[ 0 = v_{0y} - \frac{1}{2} g t \]

\[ t = \frac{2 v_{0y}}{g} \]

Substituting into the eqn for the \( x \)-direction gives

\[ 200 = v_{0x} \left( \frac{2 v_{0y}}{g} \right) \]

\[ v_{0x} = v_0 \cos(45°) = \frac{\sqrt{2}}{2} v_0 \]

\[ v_{0y} = v_0 \sin(45°) = \frac{\sqrt{2}}{2} v_0 \]

\[ 200 = \frac{2 v_{0x} v_{0y}}{g} \]

\[ 200 = \frac{2 \left( \frac{\sqrt{2}}{2} v_0 \right)^2}{g} \]

\[ 200 = \frac{v_0^2}{g} \]

\[ v_0 = \sqrt{200 g} = 44.72 \text{ m/s} \]

From the impulse momentum then

\[ M v_0 - 0 = F \Delta t \]

\[ F = \frac{M v_0}{\Delta t} = \frac{(8.46)(44.72)}{0.08} = 90.93 \text{ N} \]
Step 1: Find the speed of the block-bullet system just moment after impact using conservation of momentum.

Initial momentum: \[ P_i = MV_0 \]

Final momentum: \[ P_f = (m+M)V_1 \]

\[ MV_0 = (m+M)V_1 \]

\[ V_1 = \frac{MV_0}{m+M} \]

Step 2: Use conservation of energy to determine initial velocity.

Energy at point of impact:

\[ U_{cel} = 0 \]

\[ K = \frac{1}{2} (m+M) V_1^2 \]

Energy when spring is fully compressed:

\[ K = 0 \]

\[ U_{cel} = \frac{1}{2} k x^2 \]

Work done in between:

\[ W_e = -\frac{1}{2} M x = -M_k (m+M) g x \]

From conservation of energy:

\[ E_{total} + W = E_{total} \]

\[ \frac{1}{2} (m+M) V_1^2 - M_k (m+M) g x = \frac{1}{2} k x^2 \]

\[ \frac{1}{2} (m+M) V_1^2 = \frac{1}{2} k x^2 + M_k (m+M) g x \]

\[ V_1^2 = \frac{k x^2}{m+M} + 2M_k g x \]

\[ V_1 = \sqrt{\frac{k x^2}{m+M} + 2M_k g x} \]

\[ \frac{MW_0}{M+M} = \sqrt{\frac{k x^2}{m+M} + 2M_k g x} \]

\[ V_0 = \left( \frac{m+M}{M} \right) \sqrt{\frac{k x^2}{m+M} + 2M_k g x} \]

Plugging in the numbers:

\[ m = 0.12 \text{ kg} \]
\[ M = 0.1 \text{ kg} \]
\[ k = 150 \text{ N/m} \]
\[ x = 0.08 \text{ m} \]
\[ M_k = 0.06 \]

\[ V_0 = 2.44 \text{ m/s} \]
If \( p = 1 \) then mass = area

Mass of each strip:

\[ dm = y \, dx = \frac{1}{2} x \, dx \]

Location of each strip:

\[ x_i = x \quad y_i = \frac{y}{2} = \frac{b}{2a} x \]

\[ \Sigma \, m_i \times x_i = \int_0^a x \, dm = \int_0^a \frac{1}{2} x^2 \, dx = \frac{1}{3}a^3 = \frac{a^3}{3} \]

\[ \Sigma \, m_i = \text{total area} = \frac{1}{2} ab \]

\[ x_{cm} = \frac{\frac{1}{3}a^3}{\frac{1}{2}ab} = \frac{a}{6} \]

\[ \Sigma \, m_i \cdot y_i = \int_0^a \frac{1}{2} x \, dm = \int_0^a \left( \frac{1}{2} x \right) \, \left( \frac{1}{2} x \, dx \right) = \frac{1}{2} \int_0^a x^2 \, dx = \frac{1}{2} \cdot \frac{a^3}{3} = \frac{a^3}{6} \]

\[ y_{cm} = \frac{\frac{a^3}{6}}{\frac{1}{2}ab} = \frac{b}{3} \]

Result: for any right triangle of uniform density, the center of mass is \( \frac{a}{6} \) of the distance from the right angle in each direction.

For the triangle with corners at \((0,0), (1,1)\), and \((3,0)\),

we can split this up into two right triangles:

Triangle \( A_1 \): Area = mass = \( \frac{1}{2} (1)(1) = \frac{1}{2} \)

\[ m_1 = \frac{1}{2} \]

Center of mass: \( (x_1, y_1) = (\frac{3}{2}, \frac{1}{3}) \)

Triangle \( A_2 \): Area = mass = \( \frac{1}{2} (2)(1) = 1 \)

\[ m_2 = 1 \]

Center of mass: \( (x_2, y_2) = (\frac{5}{3}, \frac{1}{3}) \)

\[ x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{\left( \frac{1}{2} \right) \left( \frac{3}{2} \right) + (1) \left( \frac{5}{3} \right)}{\frac{1}{2} + 1} = \frac{4}{3} \]

\[ y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{\left( \frac{1}{2} \right) \left( \frac{1}{3} \right) + (1) \left( \frac{1}{3} \right)}{\frac{1}{2} + 1} = \frac{1}{3} \]

\[ (x_{cm}, y_{cm}) = \left( \frac{4}{3}, \frac{1}{3} \right) \]
The final velocities will be between those for an elastic collision (no energy loss) and a completely inelastic collision (most energy loss).

**Elastic collision:**
\[
\begin{align*}
M_1 v_{i1} + M_2 v_{i2} &= M_1 v_{f1} + M_2 v_{f2} \\
v_{i1} + v_{i2} &= v_{f1} + v_{f2}
\end{align*}
\]
\[(1) (2) + (0.5) (-3) = (1) v_{i1} + (0.5) v_{i2}
\]
\[2 + v_{i1} = -3 + v_{i2}
\]
\[5 + v_{i1} = v_{f1}
\]
\[5 = v_{f1} + 0.5 (5 + v_{i1})
\]
\[5 = 1.5 v_{f1} + 2.5
\]
\[-2 = 1.5 v_{f1}
\]
\[-\frac{4}{3} = v_{f1} \rightarrow v_{f1} = 5 - \frac{4}{3} = \frac{11}{3}
\]

**Completely inelastic collision:**
\[
\begin{align*}
M_1 v_{i1} + M_2 v_{i2} &= (M_1 + M_2) v_f \\
(1) (2) + (0.5) (-3) &= 1.5 v_f
\end{align*}
\]
\[5 = 1.5 v_f
\]
\[\frac{10}{3} = v_f
\]

For the 1 kg mass
For the 0.5 kg mass

\[\frac{-4}{3} \leq v_{i1} \leq \frac{11}{3} \quad \frac{2}{3} \leq v_{f2} \leq \frac{11}{3}
\]

0 is in the range of final velocities for the 1 kg mass, so potentially it could stop.
a) Moment of inertia of the rod:
\[ I_{rod} = \frac{1}{3} m_1 L^2 \]
Moment of inertia of the sphere:
\[ I_{sphere} = \frac{2}{5} m_2 R^2 + m_2 (L+R)^2 \]  \( \rightarrow \) from parallel axis theorem.
Total moment of inertia:
\[ I_{tot} = I_{rod} + I_{sphere} \]
\[ I_{tot} = \frac{1}{3} m_1 L^2 + \frac{2}{5} m_2 R^2 + m_2 (L+R)^2 \]

b) Center of mass of the rod: \( x_1 = \frac{L}{2} \)
Center of mass of the sphere: \( x_2 = L+R \)
\[ x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \]
\[ x_{cm} = \frac{m_1 (\frac{L}{2}) + m_2 (L+R)}{m_1 + m_2} \]

c) When moving from horizontal to vertical position, the system falls a distance of \( x_{cm} \)
From conservation of energy:
\[ (m_1 + m_2) g x_{cm} = \frac{1}{2} I_{tot} \omega^2 \]
\[ \sqrt{\frac{2 (m_1 + m_2) g x_{cm}}{I_{tot}}} = \omega \]
a) \[ v = v_0 + at \]
\[ v = 0 + 3(10) = 30 \text{ m/s} \]

b) \[ x = \frac{1}{2}at^2 = \frac{1}{2} \times 3 \times (10)^2 = 150 \text{ m} \]

c) \[ \alpha = \frac{\Delta \omega}{\Delta t} = \frac{0}{0.25} = 0 \text{ rad/s}^2 \]

d) \[ \omega = \frac{v}{r} = \frac{30}{0.25} = 120 \text{ rad/s} \]

f) The top of the tire is moving 30 m/s faster than the car, so its velocity \( \dot{v} \) with respect to the ground is \( 60 \text{ m/s} \).

f) The bottom of the tire is moving 30 m/s slower than the car, so its speed \( \dot{v} \) with respect to the ground is 0.

8) \[ v_0 = 30 \text{ m/s} \]
\[ d = 20 \text{ m} \]
\[ \Delta \theta = \frac{v_0^2}{2a} + 2a(20) \]
\[ a = -22.5 \text{ m/s}^2 \]
\[ \theta = v_0 t + \frac{1}{2}at^2 \]
\[ 0 = v_0 - at \]
\[ 0 = 30 - 22.5 \cdot t \]
\[ t = 1.33 \text{ s} \]
(9) Suppose the system falls a distance $d$.

The change in potential energy for mass $m_1$ is

$$\Delta U_1 = -m_1 g d$$

The change in potential energy for mass $m_2$ is

$$\Delta U_2 = m_2 g d \sin \theta$$

The total change in potential energy is thus:

$$\Delta U = \Delta U_1 + \Delta U_2 = -m_1 g d + m_2 g d \sin \theta$$

If we let $\theta = 0$, this means

$$U_f = m_1 g d - m_2 g d \sin 0$$

The final kinetic energy is

$$K = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} I \left(\frac{\dot{\theta}}{r^2}\right)^2$$

The energy lost due to friction is:

$$\omega_f = -\frac{\delta d = -m_2 m g d \cos \theta}{k}$$

Plugging into conservation of energy gives:

$$U_i + \Delta U + \omega_f = K$$

$$m_1 g d - m_2 g d \sin \theta - m_2 m g d \cos \theta = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} I \left(\frac{\dot{\theta}}{r^2}\right)^2$$

$$g d \left[m_1 - m_2 \sin \theta - m_2 m \cos \theta \right] = \frac{1}{2} v^2 \left[m_1 + m_2 + \frac{I}{r^2}\right]$$

$$v^2 = \frac{2 g d (m_1 - m_2 \sin \theta - m_2 m \cos \theta)}{m_1 + m_2 + \frac{I}{r^2}}$$

from kinematics $v^2 = 2a d$

$$a = \frac{v^2}{2 d}$$

$$a = g \frac{(m_1 - m_2 \sin \theta - m_2 m \cos \theta)}{m_1 + m_2 + \frac{I}{r^2}}$$
b) Using "a" for the acceleration in the positive direction is in the direction of acceleration.

\[ m_1g - T_1 = m_1a \]

\[ T_1 = m_1g - m_1a \]

\[ \text{mass } m_1 \]

\[ \begin{align*}
\text{mass } m_2 \\
T_2 \cos \theta &= \mu kg \cos \theta \\
T_2 \sin \theta &= \mu k m_2 g \cos \theta = m_2 a \\
T_2 &= m_2a + m_2g \sin \theta + \mu k m_2 g \cos \theta
\end{align*} \]

The tensions are different because the pulley is accelerating which requires a non-zero net force.

Due to the direction of the acceleration: \( T_1 > T_2 \)