Math 6A Practice problems:

Double Integrals:

1. \[\int \int e^{x^2} \, dy \, dx\]  \(\text{Hint: Switch the limits}\)

2. \[\int \int (x^2 + y^2) \, dy \, dx\]

3. \[\int \int \sqrt{1 - x^2} \, dy \, dx\]  \(\text{Try this by direct evaluation, and again by converting to polar coordinates.}\)

Triple Integrals:

Easy: \(\iiint (x+y+z) \, dV\) where \(W\) is the box on \([0,1] \times [0,2] \times [0,3]\)

Medium: Find the volume of the solid bounded by the sphere
\(x^2 + y^2 + z^2 = 12\), the cone \(z^2 = x^2 + y^2\) and the \(x\)-\(y\) plane for \(z \geq 0\)

Hard: Evaluate \(\iiint x^2 \, dV\), where \(W\) is the solid bounded by
the paraboloid \(z = x^2 + y^2\) and the plane \(2x + 3y + 4z = 12\)
Line integrals

1. Find the length of the curve $r = e^\theta$ for $0 < \theta < 2\pi$

2. Find the perimeter of the astroid $x^{2/3} + y^{2/3} = 1$

3. Evaluate $\int_C \sqrt{x^2 + y^2} \, ds$, where $C$ is the curve $x^2 - y^2 = 1$; $x > 0$ for $-1 \leq y \leq 1$

   Hint: You can parameterize this curve as $\overrightarrow{C(t)} = \left( \frac{e^t + e^{-t}}{2}, \frac{e^t - e^{-t}}{2} \right)$

4. $\overrightarrow{F} = (-y, x)$ and $C$ is the unit circle, evaluate $\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$

5. $\overrightarrow{F} = (2, y, x)$ $C$ is the triangle connecting the points $(2, 0, 0) \rightarrow (0, 3, 0) \rightarrow (0, 0, 4)$ and back to the start.

   Evaluate $\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$
Surface Integrals

1. Evaluate \( \iint_M x^2 \, dS \) where \( M \) is the part of the plane \( 2x + 4y - z = 1 \) that lies inside the cylinder \( x^2 + y^2 = 1 \).

2. Evaluate \( \iint_M \sqrt{1 - x^2 - y^2} \, dS \) where \( M \) is the part of the surface \( x^2 + y^2 + z^2 = 1 \) s.t. \( z \geq 0 \).

3. Evaluate \( \iint_A \mathbf{F} \cdot d\mathbf{S} \) where \( A \) is the surface \( z = 4 - \sqrt{x^2 + y^2} \) s.t. \( z \geq 0 \), and \( \mathbf{F} = x \mathbf{i} + z^2 \mathbf{j} + xy \mathbf{k} \).

4. This one takes a long time unless you know the divergence theorem:

\( \iint_S \mathbf{F} \cdot d\mathbf{S} \) where \( S \) is the unit sphere, and \( \mathbf{F} = x^2 \mathbf{i} + y^2 \mathbf{j} + z^2 \mathbf{k} \).