Physics 6B

Waves and Sound Examples
Wave Basics – This is a transverse* wave. 

The WAVELENGTH (\(\lambda\)) is the distance between successive wave peaks. 

The PERIOD (\(T\)) is the time it takes for the wave to move 1 wavelength. 

The FREQUENCY (\(f\)) is the reciprocal of the period. \(f = 1/T\) or \(T = 1/f\)

The main formula for all waves relates these quantities to wave speed:

\[
v = \lambda \cdot f = \lambda/T
\]

*Transverse means the wave propagates perpendicular to the displacement of the underlying medium (like waves on water or a string).
Wave Speeds

- Speed depends on mechanical properties of the medium (i.e. density or tension, etc.)

- All waves in the same medium will travel the same speed*.

- When a wave propagates from one medium to another, its speed and wavelength will change, but its frequency will be constant.

For the specific case of a wave on a string, we have a formula for speed:

\[ v_{\text{wave}} = \sqrt{\frac{\text{Tension}}{\text{mass/length}}} = \sqrt{\frac{F_T}{\mu}} \]

*We will see one exception to this later, when we deal with light (Ch. 23).
EXAMPLE  With what tension must a rope of length 2.5m and mass 0.12 kg be stretched for transverse waves of frequency 40.0Hz to have wavelength 0.75m?
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Now plug into the previous equation:

\[
30\frac{\text{m}}{\text{s}} = \sqrt{\frac{F_{\text{tension}}}{0.048\text{kg/m}}} \Rightarrow F_{\text{tension}} = 43.2\text{N}
\]
Wave Interference

**Constructive Interference:**
Waves add - larger amplitude.
These waves are “In Phase”

**Destructive Interference:**
Waves cancel - smaller amplitude.
These waves are “Out of Phase”
They are out of sync by \( \frac{1}{2} \lambda \)

Interference in action

http://www.kettering.edu/physics/drussell/Demos/superposition/superposition.html

Prepared by Vince Zaccone
For Campus Learning Assistance Services at UCSB
EXAMPLE – Two loudspeakers are placed at either end of a gymnasium, both pointing toward the center of the gym and equidistant from it. The speakers emit 256-Hz sound that is in phase. An observer at the center of the gym experiences constructive interference. How far toward either speaker must the observer walk to first experience destructive interference?
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To solve this type of problem we need to compare the distances traveled by each sound wave. If the distances differ by ½ wavelength (or 3/2, 5/2 etc.) we get destructive interference.

Label the diagram accordingly, then write down an expression for the path-length difference.
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![Diagram showing two loudspeakers and an observer at the center of the gym. The total distance is D, with distances r1 = D/2 + x and r2 = D/2 - x labeled.]

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Now when she moves over a distance \( x \), that distance is added to \( r_1 \) and subtracted from \( r_2 \).

For destructive interference we need the difference in path lengths to be a half wavelength.

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 r_1 - r_2 = \frac{1}{2} \lambda
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 r_1 - r_2 = \frac{1}{2} \lambda \Rightarrow (\frac{D}{2} + x) - (\frac{D}{2} - x) = \frac{1}{2} \lambda \Rightarrow 2x = \frac{1}{2} \lambda \Rightarrow x = \frac{1}{4} \lambda
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To solve this type of problem we need to compare the distances traveled by each sound wave. If the distances differ by $\frac{1}{2}$ wavelength (or $\frac{3}{2}$, $\frac{5}{2}$ etc.) we get destructive interference.

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To get the wavelength, use the main formula for waves: $v = \lambda f$ with $v_{\text{sound}} = 343$ m/s.

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{256 \text{ Hz}} = 1.34 \text{ m}$$

$x = \frac{1}{4} \lambda = 0.34 \text{ m}$
Standing Waves

When waves are traveling back and forth along the string, they interfere to form standing waves. These are the only waveforms that will “fit” on the string. Notice that this pattern gives us our formulas.

(a) $n = 1$

$$\frac{A}{2} = L$$

Fundamental frequency, $f_1$

(b) $n = 2$

$$2\frac{A}{2} = L$$

Second harmonic, $f_2$

First overtone

(c) $n = 3$

$$3\frac{A}{2} = L$$

Third harmonic, $f_3$

Second overtone

(d) $n = 4$

$$4\frac{A}{2} = L$$

Fourth harmonic, $f_4$

Third overtone
Standing Waves

• Basic formulas for waves on a string:

\[ \lambda_n = \frac{2L}{n} \]

\[ f_n = n \frac{v}{2L} = n \cdot f_1 \]
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• For waves in a pipe:
  • Both ends open – same as the string
  • One end closed – modified formulas
\[ \lambda_n = \frac{4L}{n} ; n = 1, 3, 5, 7, \ldots \]
\[ f_n = n \frac{v}{4L} ; n = 1, 3, 5, 7, \ldots \]
EXAMPLE A wire with mass 40g is stretched so that its ends are tied down at points 80cm apart. The wire vibrates in its fundamental mode with frequency 60Hz.
a) What is the speed of propagation of transverse waves in the wire?
b) Compute the tension in the wire.
c) What is the frequency and wavelength of the 4th harmonic?
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Since the string is in its fundamental mode (1\textsuperscript{st} harmonic) we have a formula for frequency:

\[ f_1 = 1 \cdot \frac{v}{2L} \]
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\[ f_1 = \frac{1}{2L} \cdot \nu \]

Solve this for \( \nu \):

\[ \nu = (60\text{Hz})(2 \cdot 80\text{cm}) = 9600 \frac{\text{cm}}{s} = 96 \frac{\text{m}}{s} \]

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\[ f_1 = \frac{1}{2L} \cdot \frac{v}{\mu} \]

Solve this for v:

\[ v = (60\text{Hz})(2 \cdot 80\text{cm}) = 9600\frac{\text{cm}}{s} = 96\frac{\text{m}}{s} \]

Now we can use our formula for wave speed to find the tension:

\[ v = \sqrt{\frac{F_{\text{tension}}}{\mu}}; \mu = \frac{\text{mass}}{\text{length}} = \frac{0.04\text{kg}}{0.8\text{m}} = 0.05\frac{\text{kg}}{\text{m}} \]
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To get the 4th harmonic frequency, just multiply the 1st harmonic by 4

To get the 4th harmonic wavelength, just divide the 1st harmonic by 4

\[ f_4 = 4 \cdot 60\text{Hz} = 240\text{Hz} \]

\[ \lambda_4 = \frac{2(80\text{cm})}{4} = 40\text{cm} \]
**Example:** Suppose we have 2 strings. String B has twice the mass density of string A (B is thicker and heavier). If both wires have the same tension applied to them, how can we adjust their lengths so that their fundamental frequencies are equal?
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Rearranging this equation:

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Now we can use the formula for wave speed:

\[ \frac{v_B}{v_A} = \sqrt{\frac{\mu_B}{\mu_A}} = \sqrt{\frac{1}{2}} \]

Finally we can plug this into our previous equation:

\[ L_B = \sqrt{\frac{1}{2}} \cdot L_A \]
EXAMPLE  The portion of string between the bridge and upper end of the fingerboard (the part of the string that is free to vibrate) of a certain musical instrument is 60.0 cm long and has a mass of 2.81g. The string sounds an $A_4$ note (440 Hz) when played.

● Where must the player put a finger (at what distance $x$ from the bridge) to play a $D_5$ note (587 Hz)? For both notes, the string vibrates in its fundamental mode.
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![Diagram of a string instrument with a fingerboard and bridge.]
EXAMPLE  The portion of string between the bridge and upper end of the fingerboard (the part of the string that is free to vibrate) of a certain musical instrument is 60.0 cm long and has a mass of 2.81g. The string sounds an A₄ note (440 Hz) when played.  

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When a string is vibrating in its fundamental mode (i.e. 1ˢᵗ harmonic), its wavelength is given by $\lambda = 2L$. In this case $\lambda = 1.20$m.

Now we can use our basic relationship for waves: $v = f \lambda$

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Now we work with the second case, where the finger is placed at a distance \(x\) away from the bridge. The wavelength in this case will be \(\lambda=2x\).
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Same string – same speed. Substitute into our basic formula to get:

$$528 \text{ m/s} = (587 \text{ Hz}) \cdot (2x)$$
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\[
528 \frac{\text{m}}{\text{s}} = (587\text{hz}) \cdot (2x)
\]

\[
x = 45\text{cm}
\]
Beat Frequency

Two sounds with frequencies that are similar will produce “beats”.

This is heard as a rising and falling amplitude wave with a frequency equal to the difference between the original two waves.

\[ f_{\text{beats}} = |f_2 - f_1| \]

Here is an example:

The two tones are 440Hz and 442Hz, so the beat frequency is 2Hz.

We get the same beat frequency if the tones are 438Hz and 440Hz.

Beats and more explained